A LABORATORY MANUAL

for

PHYSICS 155.3

REVISED June 2018

Department of Physics & Engineering Physics

University of Saskatchewan

Phys 155.3 Laboratory Manual

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A: <u>GENERAL INSTRUCTIONS</u>

Safety in the Laboratory

The safety issues related to the equipment and procedures used in the Physics 155 laboratory are as follows:

- electrical hazards, including potentially high current and ac line voltage (all experiments);
- trip hazards due to electrical cords hanging over the edge of the lab bench.

General Safety Policies

- ***No food or drink is to be consumed in the laboratory***
  Liquids brought into the lab room must be kept in sealed containers, which are not to be opened in the lab room and which must be placed on the floor (i.e. <u>below</u> the lab bench).

- ***Follow all instructions***
  In addition to the instructions in the laboratory manual, follow all verbal and written instructions provided by the instructional staff.

Come Prepared to the Laboratory

A proper understanding of your physics class requires a combination of studying the lecture material, solving problems, and performing experiments in the laboratory. An attempt has been made to relate the experimental work directly to the lecture material. Due to space and equipment limitations, however, experimental work may, in some cases, have to be carried out before the topic has been covered in the lectures. To compensate for this, before coming to the laboratory you should carefully read the sections of your text which are covered in the assigned experiment. If the material has been taken in lectures, the lecture notes should be studied as well. After having done this preparatory reading, carefully review the object, theory and procedure outlined in the lab manual so that you have an understanding of the purpose of the experiment, what data will be required to accomplish that purpose, and how that data will be collected. Use the material presented in this manual to plan in advance the way the experimental work should be organized.

Partners

Students will work in pairs (or groups of 3 if necessary) but each student is expected to prepare her or his own account of the work. This does not imply that partners should not cooperate; time and effort can be saved by working together in sharing the numerical calculations, checking results, and discussing the theory and performance of each experiment. Partners should share the work so that both partners understand the whole of the experiment and become familiar with the equipment. If your performance could be improved by exchanging partners, please request the demonstrator in your laboratory to arrange this, if possible.

Laboratory Schedule

A schedule of experiments will be posted online in a hallway display case near the laboratory. Each experiment normally occupies one morning or afternoon. Limited provision will be made in the schedule for the performance of those experiments which, for a legitimate reason, are not carried out at the regularly scheduled time.

Grades

Each instructor will explain the arrangements for submitting the reports, and for returning the marked reports. Each student is responsible for submitting on time a complete account of each of her/his experiments. Laboratory reports are marked by the student assistants (TAs). Note that all the assigned experiments must be completed to obtain credit in the course.

Materials

Each student should bring to the laboratory a physics laboratory notebook (coil-bound with ¼" or 5 mm grid paper), an inexpensive set of geometrical instruments and a calculator with trigonometric functions. Specific instructions about the materials required for the laboratory will be given in the first meeting of the class.

Absenteeism

Students who are absent from a laboratory period **may** be allowed to arrange with the laboratory instructor to perform the experiment some other time. If there are extenuating circumstances, such as ill health, etc., this should be discussed with the demonstrator as soon as possible.

Assistance in the Laboratory

The instructor and TA in the laboratory are there to assist you. When something arises which you do not understand, and which cannot be resolved with the help of your partner, please consult the instructor or the TA.

Before leaving the laboratory, return equipment to its original location so that the next group will not start their experiment at a disadvantage. Disconnect electrical connections and return the connecting wires to the cupboards or place them neatly on the lab bench. Turn off any gas, water, or air sources that were used during the experiment.

B: LABORATORY REPORTS

One of the aims of this laboratory is to teach you the methods that professional physicists have found satisfactory for recording the performance of experiments. Two criteria should be met by a satisfactory laboratory report:

1. A person with an educational background similar to yours should be able to easily follow through the experiment by reading your report, and if desired should be able to easily perform the experiment, obtaining results similar to yours.

2. The results of your study should stand out clearly and the conclusions drawn from these results should follow logically.

A suggested approach follows on the organization of your work to meet the above criteria. Details, such as correct procedures for drawing graphs, calculating errors, etc. are covered in later sections.

A *log book* format, rather than a *formal lab report* format will be used. The emphasis is on whether or not you understood and properly performed the experiment. The information contained in the log book should clearly indicate how the experiment was performed, what measurements were made, how these measurements were analyzed, and what conclusions were drawn. It is essential that you read the laboratory manual before coming to the lab so that you have a basic understanding of what you will be doing.

The report for each experiment will contain the following sections:

Heading
- title of experiment, date experiment performed, your name, partner's name
- overall format of report worth ½ mark

Object
- one or two sentences describing purpose of experiment
- worth ½ mark

Experiment
  Data Collection
- diagram of equipment, notes outlining procedure steps, and data, for each part of the experiment
- a brief procedure note should accompany the equipment diagram, e.g. "*The spark timer is used to record a trace of the position of the cart at equally spaced time intervals.*" This note can be written before you come to the laboratory, <u>all other procedure notes are to be done at the time the measurements are made and are to be very brief</u>. If the result of a particular step in the procedure is the measurement of a single quantity, the procedure note can be incorporated in the recording of the measurement. For example, "*distance between legs of air track, measured with tape measure*: $L = 153.2 \pm 0.4$ cm" would be a suitable procedure note. If a number of measurements are made by repetition of a single procedure, the procedure note forms a description of the corresponding table of values, e.g. "*A ruler was used to measure the displacement, Δx, of the cart in* 0.200 s *time intervals at various points along the spark trace.*"
- data should be tabulated whenever possible

- tables must be titled, and as much information as possible is to be put in the column headings rather than included with each entry. The heading of a column contains a one- or two-word descriptive label, the symbol for the quantity being tabulated, the units, the power of ten if scientific notation is being used, and the experimental error (if constant for all values in column). Individual column entries will be numbers or numbers ± errors. See Table 1 further in this introduction for an example of a proper table.
- data must be recorded, with their errors and units, directly into the lab notebook (transfer of measurements recorded on scrap paper wastes time and can introduce mistakes)
- incorrect data are to be neatly crossed out with a single line in such a way that they are still legible
- worth 3 marks

Analysis

- this section involves manipulation of the raw observational data by graphing, making numerical calculations, or in some other way specified in the instructions
- sample calculation and error calculation for each different equation
- units must be carried through calculations
- use a separate, complete page for each graph
- analysis results should be presented in tabular form whenever possible
- if possible, compare experimental results with theoretical or accepted values. The primary criterion to be used when comparing values is whether or not the values agree within experimental error (i.e. do the error ranges overlap?). Although it is not the preferred comparison criterion, percentage difference between experimental and accepted/theoretical values may be useful on occasion.
- worth 3 marks
- marks are lost in ½ mark increments for the following:
  - calculation errors
  - graph format errors
  - units not carried through calculations
  - poor presentation of results

Conclusion

- state results (with experimental error) and state whether or not there is agreement within experimental error with theoretical or accepted values
- *must include a discussion of the physical concepts examined in the experiment*
- worth 2 marks

Sources of Error

- state as many factors as you can think of which might have affected the outcome of the experiment, and which were not accounted for in any way
- explain how each of the above factors would be expected to affect your calculated results, and whether the effects would be large or small
- random measuring error need not be mentioned (it is usually accounted for in error calculations)
- the possibility of incorrectly calibrated measuring instruments should not be mentioned unless there is a particular reason for doing so. The same holds for the possibility of incorrect calculations

- worth 1 mark

NOTE: BEFORE coming to the lab the Heading and Object should be written, the equipment
          diagram should be drawn, and the Data Tables should be prepared.

Point-form may be used for the written sections of the report, but you must ensure that the
meaning of your comments is clear.

At the discretion of the marker, a report may be worth an additional 1 mark (max. mark = 10).
To receive this bonus mark the report must contain work in addition to that required by the
manual. For example, provide constructive criticism and suggestions for improvement of the
experiment or obtain additional data from which further conclusions relating to the experiment
can be drawn.

NOTE: An average lab report is worth 7.0 marks.
          A good lab report is worth 8.0 marks.
          A very good lab report is worth 8.5 to 9.0 marks.
          An excellent lab report is worth 9.0 to 10 marks.

LOG BOOK STYLE SAMPLE

The following is intended to show by example the style that is expected for your log book
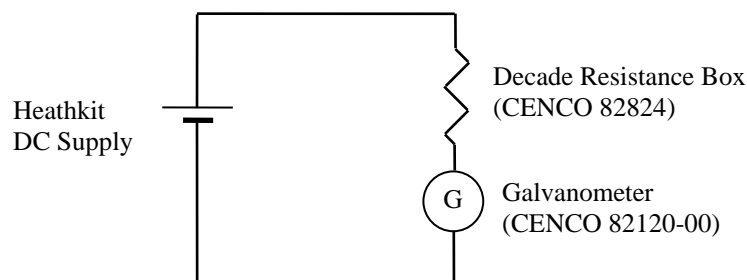reports.

15 DEC 94   DETERMINATION OF GALVANOMETER   BRIAN ZULKOSKEY
                              FULL-SCALE DEFLECTION CURRENT

Object: to determine the current required to deflect the galvanometer needle to full-scale.

Experiment:
The following circuit was connected:

Decade Resistance Box: $R = 20,000 \ \Omega \pm 1\%$

The Current Control of the Heathkit Supply was set at maximum. The Voltage Control was
initially set at minimum and was then slowly increased until the Galvanometer read full-scale
(50) when button #3 was depressed.

With the Galvanometer reading full-scale when button #3 was depressed: Heathkit Supply
Voltage = 10.20 ± 0.01 V (measured with Amprobe 37XR-A digital multimeter).

Analysis:
Calculation of Full-scale Deflection Current (galvanometer resistance ignored):

$$I = \frac{V}{R} = \frac{10.20 \text{ V}}{20,000 \text{ }\Omega} = 5.100 \times 10^{-4} \text{ A} = 510.0 \text{ } \mu\text{A}$$

Error Calculation for Full-scale Deflection Current:

$$I = \frac{V}{R} = VR^{-1}$$

$$\delta I = \delta V \cdot R^{-1} + V|(-R^{-2}\delta R)| = \frac{\delta V}{R} + V\frac{\delta R}{R^2} = \frac{0.01 \text{ V}}{20,000 \text{ }\Omega} + (10.20 \text{ V})\left(\frac{200 \text{ }\Omega}{(20,000 \text{ }\Omega)^2}\right)$$

$$\delta I = 5.6 \times 10^{-6} \text{ A} = 0.056 \times 10^{-4} \text{ A} = 5.6 \text{ } \mu\text{A}$$

The experimental value obtained for the galvanometer full-scale deflection current is $510.0 \pm 5.6 \text{ } \mu\text{A}$.

Conclusion:
The experimental value of $510.0 \pm 5.6 \text{ } \mu\text{A}$ is reasonably close to the quoted nominal value of $500 \text{ } \mu\text{A}$ for the galvanometer full-scale deflection current. Using a simple circuit and Ohm's Law enabled determination of one of the galvanometer parameters that must be known in order to use the galvanometer as a measuring instrument.

Sources of Error:
All of the errors in this procedure are expected to have small effects on the result.
 – if the galvanometer is not properly zeroed the measured full-scale current value will be too large or too small depending on which side of 0 the needle initially lies (systematic error).
 – mechanical friction in the needle suspension may cause varying needle response (random error).
 – judgement error in deciding when the meter is reading exactly full-scale (random error).
 – to account for possible variations in the meter response due to 'direction of deflection'-dependent mechanical properties, the full-scale deflection currents should have been determined for the meter deflecting to each side of 0, and the results averaged.

C: GRAPHICAL REPRESENTATION AND INTERPRETATION OF EXPERIMENTAL DATA

In order to understand a physical problem, one usually studies the dependence of one quantity upon another. Whereas sometimes such a dependence is obtained theoretically, at times it must be arrived at experimentally. The experimental determination involves obtaining experimentally a series of values of one quantity corresponding to the various arbitrary values of the other and then subjecting the data to some kind of analysis. One of the most convenient and useful means of treating the experimental data is by graphical analysis.

A graph shows the relation between the two quantities in the form of a curve. Suppose we are interested in looking at the relation between the volume of a piece of iron and its mass. If we make measurements on pieces of iron, we find that 1 cm^3 has a mass of 7.87 g, 2 cm^3 has a mass of 15.74 g, 3 cm^3 has a mass of 23.61 g, and so on. This kind of relation in which doubling the volume doubles the mass, tripling the volume triples the mass, is called direct proportion. In Physics you will encounter many cases of such relations. In describing this relation we say:

(i) mass 'is directly proportional to' volume of iron or mass 'varies directly as' the volume of iron.

(ii) Mathematically we can write the relation as $M \alpha V$, where M is the mass of a piece of iron and V is its volume and the symbol α means 'is proportional to'. If we have two different volumes of iron, V_1 and V_2, their masses M_1 and M_2 may be expressed as

$$\frac{M_1}{M_2} = \frac{V_1}{V_2}$$

Another form of this relation expresses the fact than when mass and volume are related by direct proportion, they have a constant ratio. Thus

$$\left(\frac{M}{V}\right) \text{ of one sample} = \left(\frac{M}{V}\right) \text{ of another sample} = k$$

The constant k is called the proportionality constant. In our example of iron

$$k = 7.87 \text{ g for each cubic centimetre}$$

We now express this relation as an equation for any piece of iron:

$$\left(\frac{M}{V}\right) = k \text{ or } M = kV, \text{ where the value of k as determined experimentally is 7.87 g/cm}^3.$$

An equation established in this way is an empirical equation.

We can illustrate this relation between mass and volume for iron by a graph. In trying to study the mass of a piece of iron as a function of its volume, we chose certain values for the volume and determined the corresponding values for the mass. In such a situation volume is said to be the independent variable and the mass is the dependent variable. Generally, the independent variable should be the variable that is altered in regular steps in the experiment, and the dependent variable is the quantity which is measured for each regular step of the independent variable. The independent variable (in our case the volume of a piece of iron) is plotted along the x axis and the dependent variable (in our case, the mass of the piece of iron) is plotted along the y axis. To plot the data we must choose scales – one for the vertical direction, marking off some suitable number of grams for each vertical division of the paper, and one for the horizontal

direction, marking off volumes in cm³. Now we can place a point on the graph for each pair of values that we know.

Volume, V (cm³)	Mass, M (g)
1	7.87
2	15.74
3	23.61
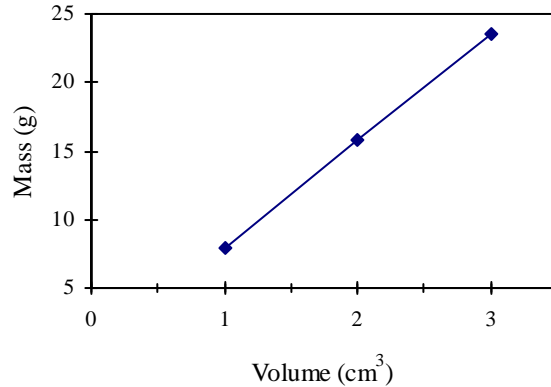
Note that a straight line can be drawn through the data points: the graph is linear. The ratio $\left(\dfrac{M}{V}\right)$ is the same for all points on the line, i.e. as already noted, $\left(\dfrac{M}{V}\right) = k$ or $M = kV$ where k is a constant. As seen here and discussed in the next section, $M = kV$ is the equation of a straight line passing through the origin if M and V are plotted against each other. k is the slope of the line.

General Linear Relationship

A straight line plot of y versus x indicates that the relationship is linear and is of the form

$$y = mx + b$$

where b is the intercept on the y-axis (the value of y when $x = 0$) and m is the slope of the line.

As an example, consider an examination of the relation between the strain (x) produced in a spring as a function of the applied force (F). A linear relation implies
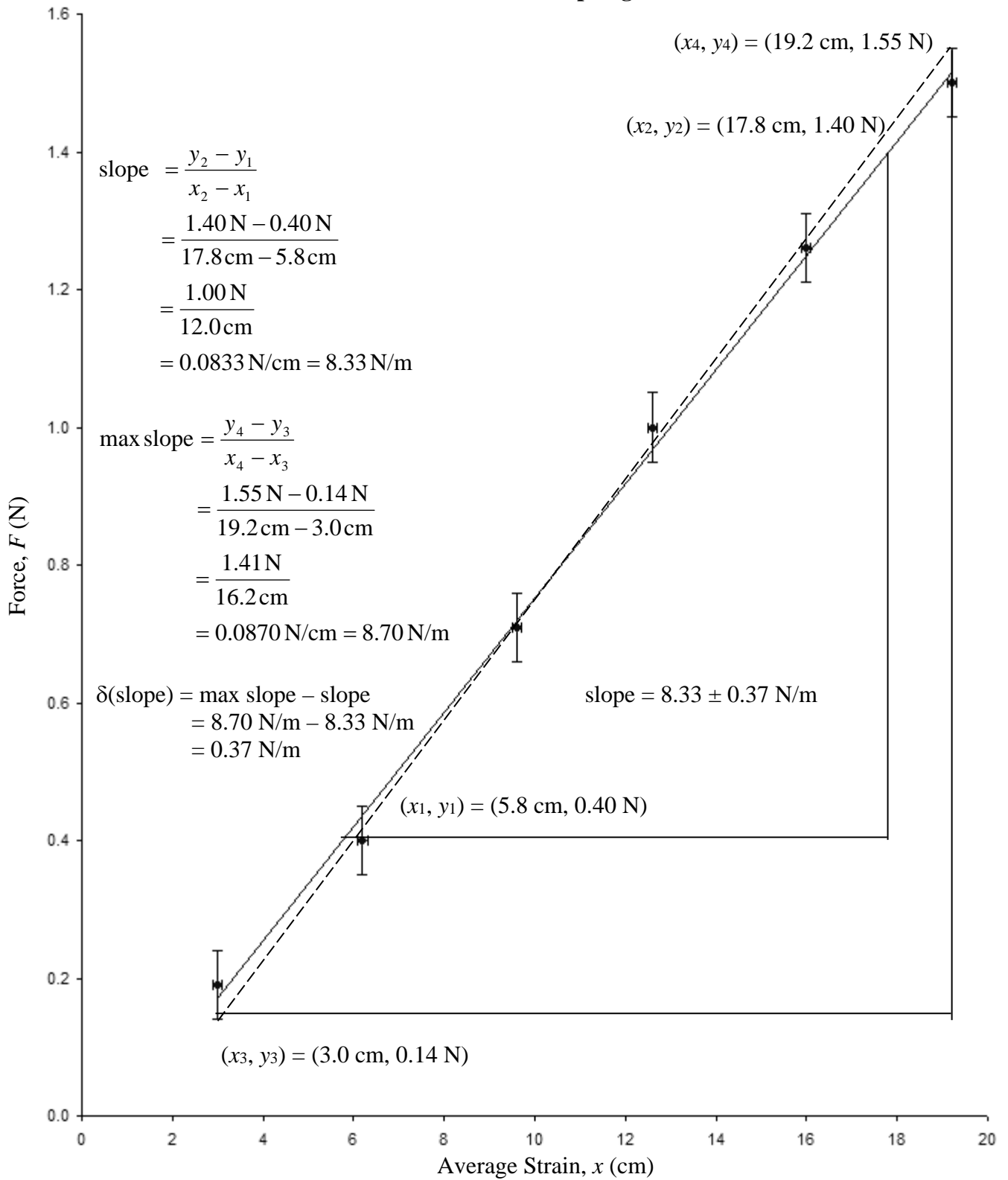
$$F = kx + b$$

where k and b are constants. For this example refer to the data in Table 1.

Table 1: Spring Constant, Spring #1

Force, F ($\pm$ 0.05N)	Average Strain, x ($\pm$ 0.1 cm)
0.19	3.0
0.40	6.2
0.71	9.6
1.00	12.6
1.26	16.0
1.50	19.2

Graph 1: Force vs. Average Strain to determine Spring Constant

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1.40\,\text{N} - 0.40\,\text{N}}{17.8\,\text{cm} - 5.8\,\text{cm}}$$

$$= \frac{1.00\,\text{N}}{12.0\,\text{cm}}$$

$$= 0.0833\,\text{N/cm} = 8.33\,\text{N/m}$$

$$\text{max slope} = \frac{y_4 - y_3}{x_4 - x_3}$$

$$= \frac{1.55\,\text{N} - 0.14\,\text{N}}{19.2\,\text{cm} - 3.0\,\text{cm}}$$

$$= \frac{1.41\,\text{N}}{16.2\,\text{cm}}$$

$$= 0.0870\,\text{N/cm} = 8.70\,\text{N/m}$$

$\delta(\text{slope}) = \text{max slope} - \text{slope}$
$= 8.70\,\text{N/m} - 8.33\,\text{N/m}$
$= 0.37\,\text{N/m}$

$\text{slope} = 8.33 \pm 0.37\,\text{N/m}$

$(x_4, y_4) = (19.2\ \text{cm},\ 1.55\ \text{N})$

$(x_2, y_2) = (17.8\ \text{cm},\ 1.40\ \text{N})$

$(x_1, y_1) = (5.8\ \text{cm},\ 0.40\ \text{N})$

$(x_3, y_3) = (3.0\ \text{cm},\ 0.14\ \text{N})$

Force, F (N)

Average Strain, x (cm)

The experimentally obtained points usually do not lie exactly on a straight line; this is expected, because experimental data are never exact. In a straight line plot of experimental results, the straight line goes above some points and below others and represents an honest attempt on the part of the plotter to show the trend of the data. See Graph 1.

If the plot of *y* versus *x* yields a straight line, the relationship is linear and is of the form

$$y = mx + b$$

If *m* = 0, *y* does not depend on *x*. If the plot is a straight line passing through the origin, *b* = 0.

If the graph directly relating the variables does not give a straight line, one may search for the form of the function by considering a few of the common types of relationships one by one.

Power Relationship

One common type of relationship is of the form

$$u = cV^n$$

where *u* and *V* are variables, *c* is a constant and *n* is the (constant) power to which *V* is raised and may be integral, fractional, positive, or negative. Taking logs of both sides gives

$$\log u = \log c + n \log V$$

Letting log *u* = *y*, log *c* = *b*, and log *V* = *x* the equation becomes

$$y = nx + b$$

where both *n* and *b* are constants. The relationship can now be plotted as a straight line. A procedure of this type is called rectification of the data before plotting. In this case, instead of plotting *u* versus *V*, the rectified data, log *u* versus log *V*, is plotted. The slope of the line obtained will give the value of the exponent *n*. Such a plot is called a log-log plot and is a general method by which an exponent may be determined.
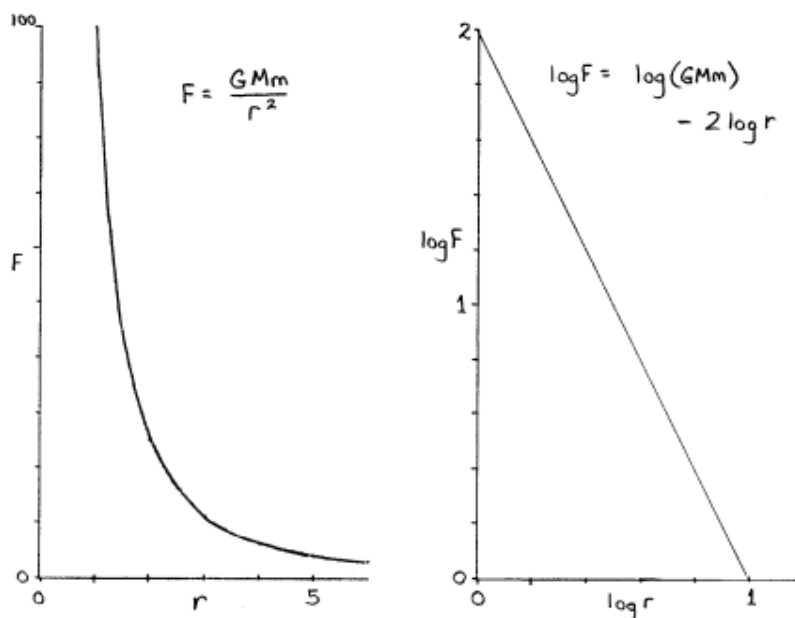
Figure 1: Rectification of a Power Relationship

Consider Newton's Law of Gravitation ($F = G\dfrac{Mm}{r^2}$). As shown in Figure 1, the plot of F versus r does not clearly show the functional relationship between F and r. An inspection of the plot of $\log F$ versus $\log r$ shows a slope of -2 and thereby indicates the inverse square relation.

Exponential Relationship

Some physical situations are described by an exponential relationship. Such relationships are most commonly written as powers of the number e ($= 2.71828...$), the base of natural logarithms.

Consider the general form of an exponential relationship:

$$u = ce^{ax} \quad \text{where } c \text{ and } a \text{ are constants.}$$

Taking logarithms to the base e (ln) of both sides of the equation gives

$$ln\ u = ln\ c + ax$$

Since c is a constant, $ln\ c$ is also a constant, and therefore a plot of $ln\ u$ versus x will be a straight line with slope a and y-intercept $ln\ c$. Such a plot is called a semi-log plot.

This type of relationship is illustrated by the following example:

The radioactive decay of a radioactive isotope is given by

$$N = N_0 e^{-\lambda t}$$

where N is the activity at time t, N_0 is the initial activity at $t = 0$, and λ is the decay constant of the isotope. As shown in Figure 2, the plot of N versus t is curved but the semi-log plot, $ln\ N$ versus t is linear, indicating that N varies exponentially with t.
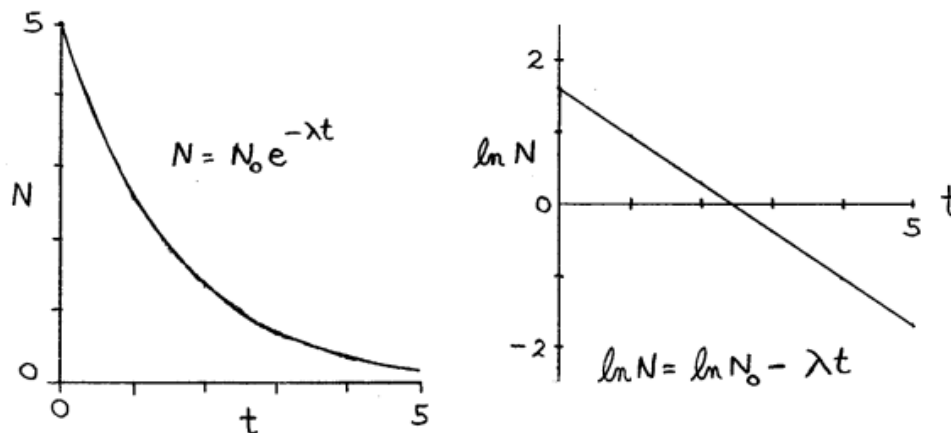
Figure 2: Rectification of an Exponential Relationship

Other situations may arise where some thought is needed to devise a plot that will yield a straight line.

It is, however, not necessary to rectify the data to check the agreement of a theory with an experiment. A graph can be plotted of the results expected on the basis of theory as applied to the physical situation under study. The experimental points can be shown on the same graph. If the theory describes the situation adequately the experimental points will lie along the theoretical curve. If, however, the experimental data points do not follow the theoretical curve then either the proposed theory is inadequate or the experiment was not designed or performed properly.

Special types of graph paper (log-log, semi-log) are available to quickly and easily test for and/or show power and exponential relationships. These papers are not marked in uniform squares. The distances along the axes are made proportional to the logarithms of the numbers that are shown on the axes. Calculators and computer plotting programs have replaced these special types of graph paper to a certain extent.

Figure 3 shows the data used in the graphs of Figure 1 (F vs. r) and Figure 2 (N vs. t) plotted on log-log and semi-log paper respectively.

The straight lines obtained indicate the power and exponential relationships respectively of these two sets of data. As discussed earlier, the slope of log F vs. log r is the value of the exponent in the power relationship. This can be determined from the log-log paper plot as follows:

$$\text{log - log slope} = \frac{\log F_2 - \log F_1}{\log r_2 - \log r_1} = \frac{\log(100) - \log(1)}{\log(1) - \log(10)} = \frac{2-0}{0-1} = -2$$

Also as discussed earlier, the slope of $ln\ N$ vs. t is the decay constant (the constant multiplying the variable in the exponent) of the exponential relationship. This can also be determined relatively easily from the semi-log paper plot.

$$\text{decay constant} = \frac{\ln N_2 - \ln N_1}{t_2 - t_1} = \frac{\ln(5) - \ln(1.8)}{0 - 5s} = -0.665\text{s}^{-1}$$

Figure 3: Use of log-log and semi-log graph paper
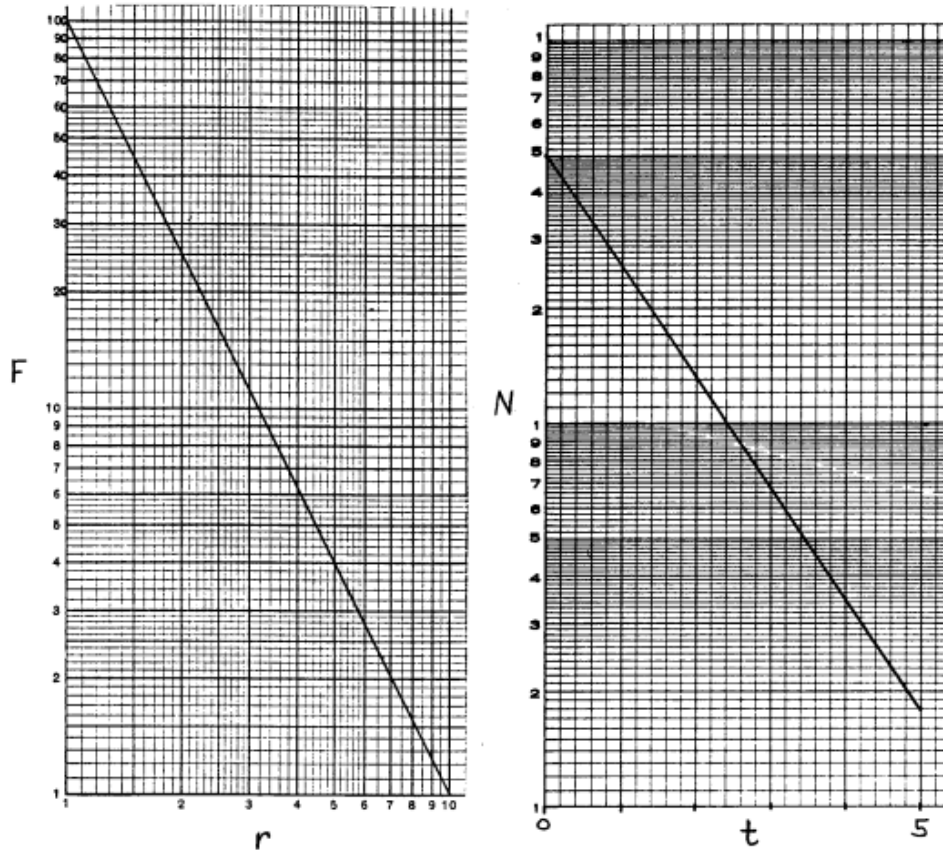
To summarize the procedure of graphical analysis of experimental data, consider two quantities y and x:

(a) If the plot of y versus x is a random scatter of points, y is not solely dependent on x in a smooth manner.

(b) If the plot of y versus x yields a straight line, the relationship is linear and is of the form $y = mx + b$. If $m = 0$, y does not depend on x. If the plot is a straight line passing through the origin, $b = 0$.

(c) If the plot of y versus x is not a straight line but log y versus log x yields a straight line then the relationship is that of a power law, the form being $y = cx^n$.

(d) If the plot of $ln\ y$ (or log y) versus x is a straight line then the relationship has an exponential form $y = ce^{ax}$. It may be necessary in practice to try plotting the log of one quantity versus the second quantity, and then the log of the second quantity versus the first.

When drawing graphs, the following rules are to be observed in order that the graphs neatly and clearly represent the experimental data.

1. Draw the axes. Axes are not normally placed along the boundaries between the graph paper and the margin although in some cases it may be necessary to do so because of the extent of the graph. Axes are usually drawn one or two large divisions in from the left and up from the bottom margin respectively. Label each axis with the variable being plotted (e.g. mass, period, time, etc.) and the units.

2. Mark the scales along the axes. The following points should be considered in choosing the scale:

    a) the graph should be easy to read and it should be possible to easily interpolate values without the need for a calculator. Make each large division equal to 1, 2, 4, 5 or 10 units. Do NOT use scales of 3, 6, 7, 9, ... units per division;

    b) the resultant curve should fill the whole graph and should not be confined to a small area of the graph paper;

    c) the geometrical slope of the curve (if it is a straight line) should be approximately unity;

    d) the point (0, 0) should not appear on the scales unless the point (0, 0) is a significant point on the graph.

3. Plot the points carefully. Mark each point so that the plotted point stands out clearly. When two sets of different data are plotted on the same set of axes use different symbols (e.g. circles and crosses) for each set of data.

4. Fit a curve to the plotted points. Most of the graphs drawn will be for the purpose of illustrating a law or determining a relationship. Hence it is reasonable to assume that the graph should be a uniformly smooth curve, possibly a straight line. Owing to the limits of experimental accuracy not all the plotted points will be exactly on the smooth curve or straight line. Use a transparent French curve or straight edge and draw a curve through the plotted points so that:

    a) the curve is smooth;

    b) the curve passes as close as possible to all the data points;

    c) when taken in moderately sized groups, as many points fall on one side of the curve as on the other.

    Note that the curve need not pass through any one point, and certainly need not pass through the end points as these end points are usually obtained at the limits of the accuracy of the

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measuring instrument. Use dashes to show the extrapolation of the curve past the range of the data points.

5. Give the completed graph a descriptive title (what is being plotted and why). This should be placed in a position where it does not interfere with the curve.

6. If a slope calculation is required it should be done on the graph page or on the page immediately following the graph page. This reduces the possibilities of error in slope calculations and makes interpretation of the graph easier. A LARGE triangle should be drawn on the graph with vertices (x_1, y_1) and (x_2, y_2) as shown in Graph 1. The vertices are then used to calculate the slope according to the formula:

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

Data points should not be used in slope calculations.

The experimental error in the slope can be calculated by performing an error calculation (discussed in a later section) on the slope equation.

7. It is sometimes desirable to indicate the experimental error in the plotted data points (see the section Discussion of Errors). This is done with ERROR BARS, which are shown in Graph 1. Vertical error bars are drawn by going above and below a data point a distance, determined by the vertical scale, corresponding to the error in the y-value. Horizontal error bars are drawn by going on either side of the data point a distance corresponding to the error in the x-value.

When error bars are drawn, the experimental error in the slope of a straight line is determined by one of the following methods:

Method 1

a) as already discussed, the best-fit straight line is drawn and its slope is calculated;

b) another line is drawn on the graph, this EXTREME-FIT line is the line passing through or near the error bars of all the data points and having the maximum or minimum possible slope;

c) the error in the best-fit slope is then taken as the absolute value of the difference between the extreme-fit slope and the best-fit slope.

Method 2

a) both the maximum-slope and the minimum-slope EXTREME-FIT lines are drawn on the graph;

b) the average slope is taken as the average of the slopes of the maximum-fit and minimum-fit lines;

c) the error in the average slope is taken as half the difference between the maximum-fit slope and the minimum-fit slope.

$$avg\ slope = \frac{(\text{max} + \text{min})}{2} \quad ; \quad \delta(avg\ slope) = \frac{(\text{max} - \text{min})}{2}$$

D: DISCUSSION OF ERRORS

Accuracy of Physical Measurements

Whether an experiment is of a precision type in which the answer is the magnitude of a quantity or of a qualitative nature in which the main aim is to substantiate a qualitative conclusion, it always involves taking measurements. Every measurement of a physical quantity is uncertain by some amount. When an observation or a result is recorded, one may ask the question - how precisely do we know this quantity? What we are asking for is a statement of some range of confidence in the quoted value or an expression of the uncertainty, sometimes called the error, in the quantity. By the word error we do not mean mistake but rather the uncertainty in the quantity.

There are three types of errors that may occur in experimental work:

1. Blunders or Mistakes on the part of the observer in reading the instruments or recording observations can be avoided by the exercise of reasonable care and therefore need no discussion. Some simple rules that will reduce the possibility of such mistakes are:
    (a) consider whether or not the reading taken is reasonable. Is it of the order of magnitude expected?
    (b) record the reading immediately and directly in the laboratory notebook. Do not attempt to remember it even for a few minutes nor record it on a loose sheet;
    (c) whenever possible have your partner check your reading.

2. Systematic Errors refer to uncertainties that influence all the measurements of a particular quantity equally. These errors may be inherent in the calibration of a particular instrument or in the particular design of the apparatus used. For example, the reading on a certain thermometer may be consistently too high because the thermometer was incorrectly calibrated. Systematic errors can usually be eliminated either by calibrating the particular instrument under conditions similar to those of the experiment or by applying a correction term in the calculations. In those cases where it is known to be necessary to apply a correction for a systematic error in the experiments performed in this laboratory, the instructors will give the details of the procedure to be followed.

3. Experimental or Random Errors remain even when all mistakes and systematic errors have been avoided or accounted for. Thus when a given measurement is repeated the resulting values, in general, do not agree exactly. No one has so far designed a perfect experiment: imperfections in our senses; in the instruments we use; and chance variations in conditions assumed to remain constant all combine to cause uncertainties in our measurements. These random errors are unavoidable and their exact magnitude cannot be determined.

    A good example of these random deviations or accidental errors is provided by the shot pattern obtained when an accurately zeroed and solidly clamped rifle is fired at a target. The pattern of fire will show a scatter of shots about the centre of fire, the number of shots in a given annular ring of constant width being progressively less as the distance from the centre of fire increases. The same effect may be demonstrated by repeatedly dropping a dart or sharp-pointed pencil vertically onto a mark on a piece of paper. The scatter of the shots about the aiming mark is analogous to the spread of experimentally obtained values about their mean value, due to accidental errors.

It is often difficult to determine the magnitude of systematic errors in an experiment. For example, in the case discussed of a systematic error introduced by a poorly calibrated thermometer one doesn't know if the error is 1° or 10° unless the thermometer is compared to a

reliable thermometer. In the case of random errors, however, the data taken in the experiment is enough to indicate how large these errors are.

A common way of approximating the magnitude of random errors is the use of significant figures. For a more reliable estimate of these errors, statistical methods and error calculations are employed.

E: SIGNIFICANT FIGURES

Numerical quantities have a different meaning in physics than they do in mathematics. The term 'significant figures' refers to the fact that while to a mathematician numbers are exact quantities, to a physicist a number is often a measured quantity and therefore inexact. A rough approximation of the uncertainty or experimental error in a number is indicated by the physicist by only writing those figures of the number in which he/she has confidence, i.e. by only writing the figures that are significant. For example, to a mathematician, 16 is the same as 16.0 or 16.00, but to a physicist, if x is a length measurement then $x = 16$ cm means 15.5 cm $< x <$ 16.5 cm; $x =$ 16.0 cm means 15.95 cm $< x <$ 16.05 cm; and $x = 16.00$ cm means 15.995 cm $< x <$ 16.005 cm.

Note that zeros used as place holders between the decimal point and the number are not significant and that zeros after the number may or may not be significant. To avoid this possible source of confusion very large and very small numbers are often written in scientific notation (also called power of ten notation). In this notation the number is written with one figure to the left of the decimal point and then multiplied by the appropriate power of ten.

e.g. $$.001804 = 1.804 \times 10^{-3} \qquad \text{(4 sig. fig.)}$$

These conventions regarding significant figures are illustrated in the following table:

Two Significant Figures	Three Significant Figures	Four Significant Figures
32	32.0	32.00
.0032	.00320	.003200
3.2×10^4	3.20×10^4	3.200×10^4

When using significant figures in calculations remember that the result of a calculation cannot be expressed to a greater degree of accuracy than the numbers from which it is calculated. For example, consider a case in which the length of an object is 20.5 cm and the width is 14.23 cm, and we are interested in finding the sum of the length and width of the object. Representing unknown figures by a question mark, we may write the sum as:

$$\begin{array}{r} 20.5?? \\ \underline{14.23?} \\ 34.7? \end{array}$$

Notice that the sum of a known figure and an unknown one is an unknown figure. The result of this example can be generalized to the following statement: the result of an addition or subtraction calculation is rounded off to the same number of DECIMAL PLACES as the number with the least decimal places that is used in the calculation. (Note that before applying this rule all the numbers involved in the calculation must be expressed to the same power of ten if scientific notation is being used.)

Similarly, when multiplying or dividing, a result can have no more SIGNIFICANT FIGURES than does the number with the fewest significant figures that is used in computing the result.

Exercises on Significant Figures

1. How many significant figures do each of the following numbers have?
    (a) 6.3×10^3 (b) .00370
    (c) 6700   (d) 670.0×10^3

2. Round off to 2 significant figures and express in scientific notation:
    (a) 3.95   (b) .02862
    (c) 219   (d) 4326

3. Calculate to the appropriate number of significant figures and express in scientific notation:
    (a) $63400 + 82$ (b) $480 \div .060$
    (c) $379 - (6 \times 10^3)$ (d) $8900 \div 30$
    (e) $290 + 6.7$ (f) $.030 - .003$
    (g) 321×3

4. Suppose that you are instructed to draw a line ten centimetres long using an ordinary ruler (one which has millimetres marked on it). How will the length of the line be best expressed?
    (a) 1×10^1 cm   (b) 10 cm   (c) 10.0 cm   (d) 10.00 cm

F: ESTIMATING ABSOLUTE ERRORS FROM RANDOM CAUSES

The ABSOLUTE error in a quantity is the estimate of the range of values to be expected due to experimental errors. The absolute error has the same units as the quantity and is denoted by the Greek symbol lower-case delta (δ) in front of the symbol for the quantity. For example, if a length measurement is denoted by x then δx is the absolute error in this length. Experimental error can also be expressed relative to the quantity. This RELATIVE error is defined as the absolute error divided by the quantity. In symbols the relative error would be $\delta x/x$ for the example mentioned above. If expressed as a percentage, the relative error becomes a PERCENTAGE error, and is calculated by $(\delta x/x) \times 100\%$.

The correct use of significant figures gives a rough estimate of the magnitude of random errors involved in taking readings. A more precise and reliable method of determining random errors is to determine absolute errors by estimation or calculation and then to use these values in subsequent calculations. Generally speaking, calculated values can be no more precise than the individual measurements. In fact, the errors accumulate so that the calculated value is usually less precise than the measurements on which it is based. The errors in a calculated quantity can be determined from the errors in each of the directly measured quantities on which the calculation is based. Before proceeding to calculate the experimental error, however, we must decide what numerical value shall be given to the absolute error in any measurement. The following rules may be used for assigning absolute errors:

1. When a quantity is just measured once, the measured value is taken as being the 'true' value and the absolute error due to random causes is estimated as the MAXIMUM INSTRUMENTAL ERROR in the measuring instrument. The maximum instrumental error will usually be the least count of the instrument (the size of the smallest scale division). For example in measuring the length of an object with a meter stick, the error would be ±1 mm since 1 mm is the smallest scale division on the meter stick.

2. A more accurate estimate of error can be obtained when a quantity is measured several times. The effect of random errors is usually to cause measurements to be distributed according to a normal curve (also called a bell-shaped or a Gaussian curve). An example of such a distribution would be obtained by a person shooting billiard balls at a meter stick placed at the edge of a billiard table. If he/she aimed at the 50 cm mark then his/her record of hits after many trials might look like the following:
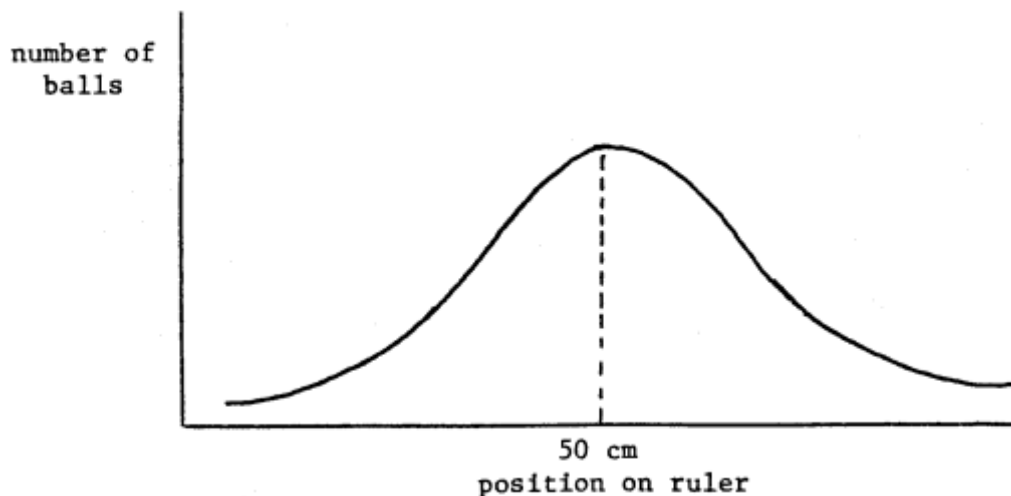
This bell-shaped distribution is caused by random errors: errors such as irregular wind currents, lack of sphericity in the billiard balls, etc. Note that the effect of a systematic error would be quite different. If the table was not level, for example, one might obtain a bell-shaped distribution around the 55 cm mark but the shape of the curve would be unchanged.

It is beyond the scope of this class to enter into a complete discussion of the statistical methods but the main results of the analysis of normal curves are as follows:

(a) The 'best' or most probable value in a normal distribution is the arithmetic mean $\bar{x}$

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

where x_1, x_2, ..., x_n are the individual measurements and n is the number of measurements.

(b) To provide a measure of the error in the measurement, one defines a quantity σ called the STANDARD DEVIATION IN THE MEAN:

$$\sigma_x = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

For a normal distribution the probability of obtaining an error in x less than $\pm\sigma$ is 68.2%. In other words, on an average of 68 times out of 100 the true value of the quantity being measured will fall within the range of $\bar{x} - \sigma$ to $\bar{x} + \sigma$. The probability of an error less than $\pm 2\sigma$ is 95.4% and the probability of an error less than $\pm 3\sigma$ is 99.7%.

Note that this method of treating errors has the following limitations: it assumes a normal distribution which is usually but not always valid; it provides no indication of errors due to systematic causes; the above formula is not the same as the formula used for standard deviation on most calculators.

The suggested procedure for this course will be to quote the error as $\bar{x} \pm \sigma$ with 68% confidence. However, if the instrument error as described in 1. is greater than σ, then the instrument error should be taken to be the absolute error.

G: PROPAGATION OF ERRORS IN CALCULATIONS

In a number of cases the quantity of interest cannot be measured directly – it is calculated by using two or more directly measured quantities. When uncertain quantities are used in numerical calculations one usually wishes to know the error in the result. The simplest method, although not the most accurate, is to determine the maximum possible error.

MAXIMUM POSSIBLE ERROR METHOD

CALCULUS TECHNIQUE

One technique for obtaining an error expression is to make use of calculus. If you are familiar with the rules for derivatives you will find this technique relatively simple.

Bottom Line:
The maximum experimental uncertainty in a calculated value is obtained by adding the absolute values of the terms arising from the partial differential of the expression for the calculated value, treating each measured value as a variable.

Details:
Before proceeding, a brief review of some of the definitions and rules of differential calculus will be helpful.

Suppose y is some function of the variable x. Then the derivative of y with respect to x is dy/dx, sometimes denoted y'. Mathematicians speak of 'd/dx' as being the differentiation operator: '$d(\ )/dx$' means 'take the derivative with respect to x' of whatever function is placed in the parentheses. '$d(\ )/dx$' can also be considered to represent the small change $d(\ )$ that occurs in ( ), for a small change dx in x, divided by this small change in x.

Therefore, starting from

$$\frac{dy}{dx} = y'$$

and multiplying both sides by the small change dx in x, yields

$$dy = y'\, dx$$

Therefore the small change in the variable y arising from a small change in the variable x is calculated by multiplying the small change in x by the derivative of y with respect to x.

The 'small changes' are called the DIFFERENTIALS of the appropriate variables.

The rules of calculus for calculating differentials are the same as those for calculating derivatives.

Suppose $y = cx^n$,

Using the exponent rule, the DERIVATIVE of y with respect to x, $dy/dx = y'$, is

$$\frac{dy}{dx} = y' = ncx^{n-1}$$

The DIFFERENTIAL of y is

$$dy = ncx^{n-1}\, dx$$

Similarly, if u, v, and w are functions of some (dummy) variable x:

$$\frac{d}{dx}(u+v-w) = \frac{du}{dx} + \frac{dv}{dx} - \frac{dw}{dx} \qquad \text{(derivative)}$$

$$d(u+v-w) = du + dv - dw \qquad \text{(differential)}$$

$$\frac{d}{dx}(uvw) = vw\frac{du}{dx} + uw\frac{dv}{dx} + uv\frac{dw}{dx} \qquad \text{(derivative)}$$

$$d(uvw) = vw\,du + uw\,dv + uv\,dw \qquad \text{(differential)}$$

$$\frac{d}{dx}\left(u/v\right) = \frac{v(du/dx) - u(dv/dx)}{v^2} \qquad \text{(derivative)}$$

$$d\left(u/v\right) = \frac{v\,du - u\,dv}{v^2} \qquad \text{(differential)}$$

$$\frac{d}{dx}(\sin\theta) = \cos\theta\,\frac{d\theta}{dx} \qquad \text{(derivative)}$$

$$d(\sin\theta) = \cos\theta\,d\theta \qquad \text{(differential)}$$

Application of these rules of calculus to calculation of errors proceeds as follows:

1. Ensure that the original expression is in its simplest form and that each variable is independent of all other variables. You may also find it easier if you replace division with multiplication and negative exponents. e.g. replace a/b with ab^{-1}.

2. Calculate the differential of the expression. (Each symbol representing a measured value is taken as a variable to be differentiated.)

3. Take the absolute value of each term in the differential expression. A TERM is defined as an expression containing ONE differential quantity. e.g. *(a/b) dx*, *uvw dx*, *y dv*, *(a–b) dx*. The expression *(dx – dy)* is TWO terms:

$$(dx - dy) \;=\; dx - dy \;=\; dx + (-dy)$$

The effect of this rule is that *(dx – dy)* becomes

$$|dx| + |-dy| \;=\; dx + dy$$

i.e. The terms in the differential expression add.

4. Whenever the differential of any variable appears, substitute the absolute error of that variable.

5. When angles are used, the absolute error of the angle must be expressed in radians (not degrees).

The following examples illustrate this technique.

Example 1

The equation for kinetic energy is $KE = \frac{1}{2}mv^2$ where m and v are mass and speed respectively. The differential of KE is:

$$d\mathrm{KE} \;=\; \tfrac{1}{2}dm\,v^2 \;+\; \tfrac{1}{2}m\,2v\,dv$$

Thus the absolute error of K is:

$$\delta\mathrm{KE} \;=\; \tfrac{1}{2}\delta m\,v^2 \;+\; \tfrac{1}{2}m\,2v\,\delta v$$

where δm and δv are the absolute errors in m and v.

Example 2

The equation for centripetal acceleration is $a = \dfrac{v^2}{R}$, where R and v are the trajectory radius and speed of the object undergoing centripetal acceleration. Writing the equation as $a = v^2 R^{-1}$, the differential of a is:

$$da = 2v \, dv \, R^{-1} + v^2 (-R^{-2}) \, dR$$

Applying rules 3 and 4 yields

$$\delta a = 2v \, \delta v \, R^{-1} + v^2 R^{-2} \, \delta R$$

where δv and δR are the absolute errors in v and R.

NON-CALCULUS TECHNIQUE

Bottom Line:
The maximum <u>absolute</u> uncertainty in the <u>addition or subtraction</u> of measured values is the sum of the absolute errors in the measured values. The maximum <u>percentage (relative)</u> uncertainty in the <u>multiplication or division</u> of measured values is the sum of the percentage (relative) errors in the measured values. The maximum <u>percentage (relative)</u> uncertainty in <u>a measured value raised to a power</u> is the product of the power and the percentage (relative) error in the measured value.

Details:

Addition and Subtraction

Consider the sum of two measurements, 25.4 ± 0.1 cm and 7.5 ± 0.2 cm. The result is 32.9 cm. To find the error in this result, consider that the first measurement lies between 25.3 and 25.5 cm. The second measurement lies between 7.3 and 7.7 cm. Therefore the sum lies between 32.6 and 33.2 cm, which may be written as 32.9 ± 0.3 cm. The absolute error in the result is the sum of the absolute errors of the two measurements. In symbolic form this may be written:

$$(a \pm \delta a) + (b \pm \delta b) = (a + b) \pm (\delta a + \delta b)$$

To generalize:

if $$P = Q_1 + Q_2 + ... + Q_n$$

where $Q_1, Q_2, ... Q_n$ are all measured quantities,

then $$\delta P = \delta Q_1 + \delta Q_2 + ... + \delta Q_n$$

Now consider the subtraction of the two measurements from the previous discussion. The result is 25.4 cm – 7.5 cm = 17.9 cm. The limits of the range of the result are obtained by calculating

$$(25.4 \text{ cm} + 0.1 \text{ cm}) - (7.5 \text{ cm} - 0.2 \text{ cm}) = 25.5 \text{ cm} - 7.3 \text{ cm} = 18.2 \text{ cm}$$

and

$$(25.4 \text{ cm} - 0.1 \text{ cm}) - (7.5 \text{ cm} + 0.2 \text{ cm}) = 25.3 \text{ cm} - 7.7 \text{ cm} = 17.6 \text{ cm}$$

Thus the result is expressed as 17.9 ± 0.3 cm. Thus when quantities are subtracted the absolute error in the result is the sum of the absolute errors in the quantities.

Combining this with the result for addition, if P is calculated from the addition and/or subtraction of a number of measured quantities Q_i,

$$P = Q_1 + Q_2 - Q_3 - Q_4 + ... + Q_n$$

then $$\delta P = \delta Q_1 + \delta Q_2 + \delta Q_3 + \delta Q_4 + ... + \delta Q_n$$

WHEN QUANTITIES ARE ADDED OR SUBTRACTED THE MAXIMUM POSSIBLE ABSOLUTE ERROR IN THE RESULT IS THE SUM OF THE ABSOLUTE ERRORS IN THE QUANTITIES.

Multiplication

Consider a rectangle measured to have sides $a \pm \delta a$ and $b \pm \delta b$. The area A and its absolute error δA are calculated as follows:

$$A \pm \delta A = (a \pm \delta a)(b \pm \delta b) = ab \pm (\delta a)b \pm a(\delta b) \pm (\delta a)(\delta b)$$

Factoring the product ab yields:

$$A \pm \delta A = ab\left[1 \pm \frac{\delta a}{a} \pm \frac{\delta b}{b} \pm \frac{(\delta a)(\delta b)}{ab}\right]$$

Noting that $ab = A$, and ignoring the second order term $(\delta a)(\delta b)/ab$ since it will be small compared to the other terms in the square brackets, yields:

$$\delta A = A\left[\frac{\delta a}{a} + \frac{\delta b}{b}\right]$$

i.e. $$\frac{\delta A}{A} = \frac{\delta a}{a} + \frac{\delta b}{b}$$

In general, if P is the product of measured quantities $Q_1, Q_2, ... Q_n$, then the <u>relative</u> error in P is

$$\frac{\delta P}{P} = \frac{\delta Q_1}{Q_1} + \frac{\delta Q_2}{Q_2} + ... + \frac{\delta Q_n}{Q_n}$$

THE MAXIMUM POSSIBLE RELATIVE ERROR IN A PRODUCT IS THE SUM OF THE RELATIVE ERRORS IN THE FACTORS. Noting that relative and percentage errors are related by % error = $100 \times$ (relative error), the maximum possible percentage error in a product is the sum of the percentage errors in the factors.

Division

Let P be the result of the division of Q by R.

i.e. $$P = \frac{Q}{R}$$

Then

$$P \pm \delta P = \frac{Q \pm \delta Q}{R \pm \delta R} = \frac{Q\left[1 \pm \frac{\delta Q}{Q}\right]}{R\left[1 \pm \frac{\delta R}{R}\right]} = \left(\frac{Q}{R}\right)\frac{1 \pm \frac{\delta Q}{Q}}{1 \pm \frac{\delta R}{R}}$$

Applying the binomial expansion and ignoring 2nd order terms yields:

$$P \pm \delta P = \left(\frac{Q}{R}\right)\left[1 \pm \frac{\delta Q}{Q} \pm \frac{\delta R}{R}\right] = P\left[1 \pm \frac{\delta Q}{Q} \pm \frac{\delta R}{R}\right]$$

so

$$\delta P = P\left[\frac{\delta Q}{Q} + \frac{\delta R}{R}\right]$$

i.e.

$$\frac{\delta P}{P} = \frac{\delta Q}{Q} + \frac{\delta R}{R}$$

THE MAXIMUM POSSIBLE RELATIVE ERROR IN A QUOTIENT IS THE SUM OF THE RELATIVE ERRORS IN THE DIVISOR AND DIVIDEND.

Powers and Roots

Using the result obtained for multiplication and realizing that the expression Q^n means to multiply Q by itself n times, if

$$P = Q^n$$

then

$$\frac{\delta P}{P} = n\frac{\delta Q}{Q}$$

THE MAXIMUM POSSIBLE RELATIVE ERROR IN RAISING A BASE VALUE TO AN EXPONENT IS THE PRODUCT OF THE EXPONENT AND THE RELATIVE ERROR IN THE BASE.

For a more general example, consider the quantity N given by the following expression:

$$N = (AB^4)/(CD^{1/2})$$

where *A, B, C, D* are all measured quantities. The maximum possible relative error in N is

$$\frac{\delta N}{N} = \frac{\delta A}{A} + 4\frac{\delta B}{B} + \frac{\delta C}{C} + \tfrac{1}{2}\frac{\delta D}{D}$$

Note that the error in N is influenced more by errors made in measuring B than any of the other quantities. In designing and performing such an experiment great care should be taken to measure B with the highest degree of precision attainable.

SUMMARY

1. All measured quantities have associated with them an uncertainty which can be expressed as an absolute or relative error and which limits the number of significant figures of these quantities and any results calculated from them.

2. When uncertain quantities are added or subtracted the maximum possible absolute error of the result is determined by adding the absolute errors of the separate quantities.

3. When uncertain quantities are multiplied or divided the maximum possible relative error of the result is determined by adding the relative errors of the separate quantities.

4. When an uncertain quantity is raised to a power (including a fractional power) the maximum possible relative error in the result is determined by multiplying the relative error in the quantity by the exponent.

5. When uncertain quantities are related in more complicated ways (e.g. log functions, trigonometric functions) the result must be calculated twice, once with the experimental values and then with the extreme values of the quantities. The maximum possible absolute error is the difference between these two values of the complicated function.

ADDITION IN QUADRATURE METHOD

For the purposes of this laboratory, it is sufficient to use the maximum possible error as the error in the result of a calculation. The following discussion of the addition in quadrature method is included for completeness and because it is the method most often used by scientists.

Because of the assumed random nature of experimental errors the maximum possible error overestimates the error in the result of a calculation. It is unlikely that all the errors in the uncertain quantities will be in the same 'direction', which is what is assumed in the maximum possible error method. A more accurate estimate of the propagation of errors through calculations is obtained by adding the errors in quadrature: the 'sum of the errors' is replaced by the 'square root of the sum of the squares of the errors'.

Thus the absolute error in the result of addition and/or subtraction of uncertain quantities is the square root of the sum of the squares of the absolute errors in the uncertain quantities. If

$$P = Q_1 + Q_2 + Q_3 + \ldots + Q_n$$

then

$$\delta P = \sqrt{\sum_i (\delta Q_i)^2}$$

Similarly, the relative error in the result of multiplication and/or division of uncertain quantities is the square root of the sum of the squares of the relative errors in the uncertain quantities. If

$$P = QR/T$$

then

$$\frac{\delta P}{P} = \sqrt{\left(\frac{\delta Q}{Q}\right)^2 + \left(\frac{\delta R}{R}\right)^2 + \left(\frac{\delta T}{T}\right)^2}$$

Powers and roots are handled in a similar manner. If

$$N = (AB^4)/(CD^{1/2})$$

then

$$\frac{\delta N}{N} = \sqrt{\left(\frac{\delta A}{A}\right)^2 + \left(4\frac{\delta B}{B}\right)^2 + \left(\frac{\delta C}{C}\right)^2 + \left(\frac{1}{2}\frac{\delta D}{D}\right)^2}$$

ERROR CALCULATION QUESTIONS

These questions are designed to help you in reviewing error calculations. In each of the questions you will be given an equation and some data to use in the equation. Each question consists of four parts.

(a) Derive the error equation. The error equation should be in symbols only.

(b) Use the data to do the calculation. Watch your significant figures and the units.

(c) Calculate the absolute error.

(d) Express the answer, with its absolute error, to the appropriate number of significant figures.

SAMPLE

The equation for kinetic energy is $KE = \frac{1}{2}mv^2$ where m and v are mass and speed respectively. Suppose that the following values were obtained in an experiment:

$m = 5.32 \pm 0.03$ kg and $v = 2.37 \pm 0.07$ m/s

(a) $\delta KE = \frac{1}{2}\delta m\, v^2 + \frac{1}{2}m\, 2v\, \delta v$ where δm and δv are the absolute errors in m and v.

(b) $KE = \frac{1}{2}(5.32 \text{ kg})(2.37 \text{ m/s})^2 = 14.94 \text{ kg·m}^2/\text{s}^2 = 14.9 \text{ J}$

(c) $\delta KE = \frac{1}{2}(0.03 \text{ kg})(2.37 \text{ m/s})^2 + \frac{1}{2}(5.32 \text{ kg})(2)(2.37 \text{ m/s})(0.07 \text{ m/s}) = 0.967 \text{ J}$

(d) $KE = 14.9 \pm 1.0$ J

1. The equation of an ellipse is

$$y = b\sqrt{1 - \frac{x^2}{a^2}},$$

$a = 22.6 \pm 0.1$ cm, $b = 46.9 \pm 0.1$ cm, $x = 12.2 \pm 0.1$ cm: **$y = 39.48 \pm 0.29$ cm**

2. From Kepler's third law of planetary motion, the orbital period T of a satellite in a circular orbit at height h above a massive body of radius R and mass M is:

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM}},$$

$R = (1.738 \pm 0.001) \times 10^6$ m; $h = (1.1 \pm 0.1) \times 10^6$ m

$G = 6.67 \times 10^{-11}$ N·m²/kg²; $M = (7.35 \pm 0.01) \times 10^{22}$ kg

$T = (1.357 \pm 0.073) \times 10^4$ s

3. Consider an Atwood's machine in which the pulley is neither frictionless nor massless. In that case the acceleration is

$$a = \frac{(m_2 - m_1)g}{(I/R^2) + m_1 + m_2 + m}$$

where I is the moment of inertia of the pulley, R is its radius and m is a mass that is used to counteract the friction. Given $g = 980$ cm/s², $m_1 = 100 \pm 1$ g, $m_2 = 200 \pm 2$ g, $I = 950 \pm 50$ g·cm², $R = 3.5 \pm 0.1$ cm and $m = 5.0 \pm 0.1$ g: **$a = 256 \pm 15$ cm/s².**

4. When resistances R_1, R_2, and R_3 are connected in parallel the total resistance, R, is given by the equation

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Given: $R_1 = 50 \pm 5 \ \Omega$, $R_2 = 70 \pm 5 \ \Omega$, $R_3 = 95 \pm 5 \ \Omega$. **$R = 22.3 \pm 1.8 \ \Omega$**

H: SAMPLE CALCULATIONS

For each different equation used in the analysis of an experiment a sample calculation (and possibly a sample error calculation) are required. The following discussion explains what is required in a proper sample calculation.

1. Label the calculation descriptively. Explicitly state, **in words**, what is being calculated. If this is one of many similar calculations, specify which one.
2. State the values, with their units and experimental errors, that will be used in the calculation.
3. State the equation in symbols.
4. Show the substitution of the values (with their units but *not* their experimental errors) into the equation.
5. Give the result of the calculation. (**Don't round off**, keep extra significant figures at this stage, especially trailing zeros. Rounding will be done later).
6. State the error equation in symbols.
7. Show the substitution of values (with units) into the error equation.
8. Give the unrounded result of the error calculation, with units.
9. State the result and its absolute error with the appropriate number of significant figures.

The following rule is to be used in rounding off the results of calculations: **ROUND OFF THE ABSOLUTE ERROR IN THE RESULT TO TWO SIGNIFICANT FIGURES, THEN ROUND OFF THE RESULT TO THE SAME NUMBER OF DECIMAL PLACES AS THE ABSOLUTE ERROR.** If scientific notation is being used, express both the result and its absolute error to the same power of ten before applying the second part of this rule.

As an example, suppose the mass and speed of a car on an air track have been measured and the magnitude of the car's momentum is to be calculated. The following is the correct sample calculation:

1. Sample Calculation of Momentum of Car   *(label)*
   on Air Track for Trial #1

2. $m = 0.675 \pm 0.001$ kg   *(statement of values)*
   $v = 0.123 \pm 0.005$ m/s

3. $p = mv$ *(equation in symbols)*
4. $p = (0.675$ kg$)(0.123$ m/s$)$ *(substitution of values)*
5. $p = 0.083025$ kg·m/s   *(calculation result)*

6. $\delta p = |\delta m\ v| + |m\ \delta v|$ *(error equation in symbols)*

7. $\delta p = (0.001$ kg$)(0.123$ m/s$) + (0.675$ kg$)(0.005$ m/s$)$ *(substitution of values)*

8. $\delta p = 0.003498$ kg·m/s $= 0.0035$ kg·m/s   *(error calculation result)*

9. $p = 0.0830 \pm 0.0035$ kg·m/s **or**   *(round off)*
   $p = (8.30 \pm 0.35) \times 10^{-2}$ kg·m/s
   if scientific notation is being used

I: COMPARING TWO QUANTITIES

Experiments sometimes involve determining a physical quantity by two different methods to see if the results of the two methods agree. Sometimes one wishes to compare the experimental results with the predictions of a theory. In either case it is highly unlikely that there will be exact numerical agreement between the values to be compared. To compare the quantities, and decide if there is agreement, their experimental errors must be calculated. If the ranges of the two values overlap the values are said to agree within the limits of the error of that experiment. If the ranges do not overlap, then the values do not agree.

For example, if one determines the value of g to be 979 ± 3 cm/s^2 and the known value is 981 cm/s^2 then the two values agree because the range of the experimental value (976 cm/s^2 to 982 cm/s^2) includes the known value.

Bar Graph Comparison

$$g_{\text{exp}} = 979 \pm 3 \text{ cm/s}^2$$

g_{acc}

| 975 | 976 | 977 | 978 | 979 | 980 | 981 | 982 | 983 cm/s^2 |

range of g_{exp} values
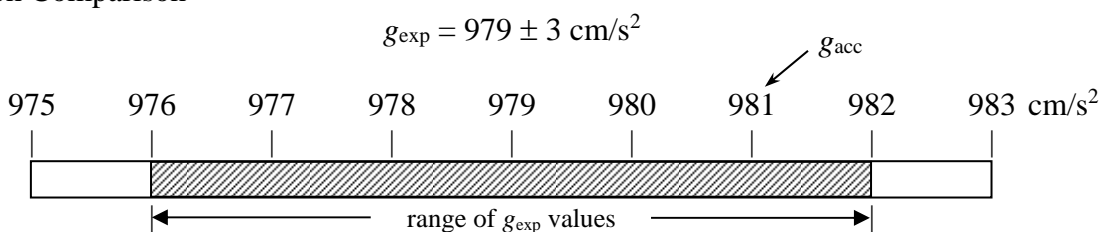
Algebraic Comparison

The test for error range overlap can be expressed quantitatively as follows. Let $x_1 \pm \delta x_1$ and $x_2 \pm \delta x_2$ be two values that are to be tested for agreement within experimental error. If the magnitude of the difference between the values is less than the sum of their absolute errors then the values agree.

i.e.   if $|x_1 - x_2| \leq \delta x_1 + \delta x_2$

then x_1 and x_2 are said to be equal within experimental error.

When experimental errors have not been computed, the experimental value is compared with the standard known value by computing the percentage difference:

$$\% \text{ difference} = \left| \frac{\text{experimental value} - \text{known value}}{\text{known value}} \right| \times 100\%$$

When a physical quantity does not have an accepted value, the following equation is used to calculate the percentage difference between values determined by two experimental methods:

$$\% \text{ difference} = \frac{\text{larger value} - \text{smaller value}}{\text{smaller value}} \times 100\%$$

When the percentage change in a value is to be calculated, the following equation is used:

$$\% \text{ change} = \frac{\text{final value} - \text{initial value}}{\text{initial value}} \times 100\%$$

P1   <u>**PROBLEM LABORATORY ON FORCE FIELDS**</u>

Created January 2006 by Professor E. Salt

Revised July 2009 and May 2018 by Brian Zulkoskey

PURPOSE

The purpose of this laboratory is to investigate the relationship between force and energy in a conservative field and to understand the differences between conservative and non-conservative fields.

LAB EQUIPMENT

The laboratory requires a lab notebook, a calculator, and a straightedge ruler.

USEFUL RELATIONSHIPS

$W_{by\ field} = -(\Delta E_{field}) = -(E_{field\ final} - E_{field\ initial})$; $W_{by\ mover} = -W_{by\ field}$; $W = \vec{F} \cdot \vec{d}$; $\Delta PE_{grav} = mg\Delta h$

QUESTIONS

1. Figure 1 is a map showing force lines and equal energy contours for a plane in a conservative field. The energy contours have units of J/kg. This means that the amount of energy transferred to/from the field as an object is moved depends on the mass of the object. This also means that the force acting on an object depends on the mass of the object.

    (a) If an object is moved from a position where its centre of mass is at point A to a position where its centre of mass is at point B, is energy transferred to or retrieved from the field (i.e. does the <u>mover</u> do positive or negative work on the object in moving it from point A to point B)? If the mover does positive work, then energy is being transferred to the field.

    (b) How much energy is transferred to the field if an object that has a mass of 1 kg is moved from a position where its centre of mass is at point A to a position where its centre of mass is at point B?

    (c) How much energy is transferred to the field if a 1 kg object is moved from a position where its centre of mass is at point D to a position where its centre of mass is at point A?
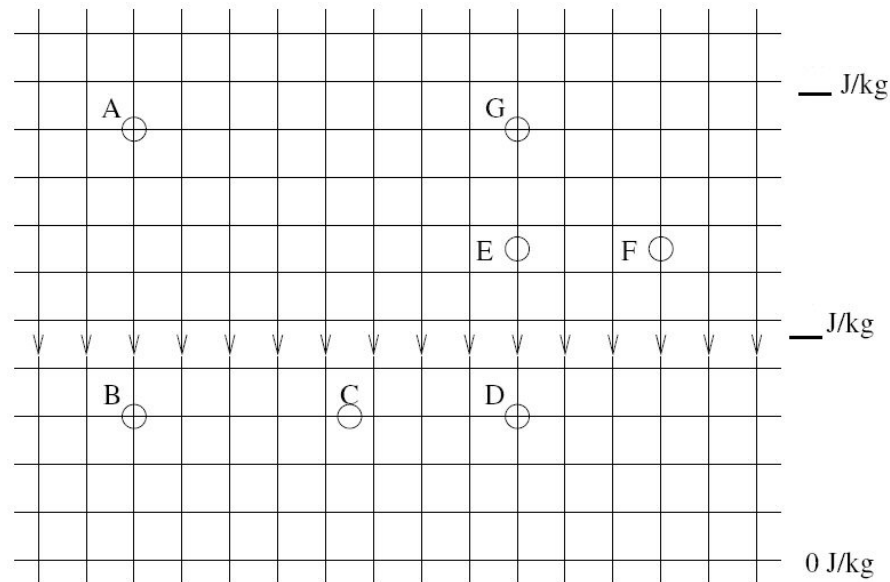
Figure 1: Conservative Force Field with Energy contours in steps of __ J/kg

(d) Construct a table as shown in Table 1. Determine the amount of energy that the mover must supply (i.e. the amount of energy transferred to the field) to move a 1 kg mass between the points listed in Table 1.

(e) A force field is conservative if and only if it takes 0 J of energy to move an object along any path that ends where it started. Since Figure 1 is a map of a conservative field it should require a net of 0 J of energy to move a 1 kg object along the path A-B-C-D-E-F-G-A. Verify that this is true using your values from Table 1.

(f) Assume that the distance between neighbouring energy contours in the map of Figure 1 is 1 meter. What is the magnitude of the force exerted by the field on a __ kg mass located at point A? (Recall that the average force on an object in a force field is the amount of energy transferred to the object from the field divided by the distance moved in the direction of the force field.)

(g) What is the magnitude of the force exerted by the field on a __ kg mass located at point C?

(h) Does the strength of the field (i.e., the force) depend on the location of the mass?

(i) A 1 kg mass is moved in a straight line from point D to point A. Assume the distance between energy contours and the distance between force lines are both 1 m.

   i. Over what distance is the object moved?

   ii. What is the magnitude of the component of the force due to the field that <u>directly</u> opposes the motion?

   iii. Does the work done to move an object from point D to A equal the directly-opposing force times the distance between points D and A?

   iv. What is the magnitude of the component of the force that acts at a right angle to the direction of movement?

Table 1: Energy table for moving a 1 kg mass

| move 1 kg object | | Energy spent by mover |
From	to	to overcome force due to field
A	B	
B	C	
C	D	
D	E	
E	F	
F	G	
G	A	

2. Consider an air hockey table that is 1 meter wide by 2 meters long with the length of the table running north-south (i.e. the goals are due north/south of each other). The table is not mounted level. The north end of the table is __ cm higher than the south end so that if the puck is placed in the middle of the table and released it will accelerate due south and end up in the centre of the south end goal. Assume that there is no friction between the table and the puck.

Draw an energy-contour/force-field map for the surface of the air hockey table. Such a map shows the energy contours and also shows the lines of force that act on a puck in the plane of the table's surface. Draw five energy contours (bottom, ¼, ½, ¾, and top) and four lines of force. The map should show the energy levels on a per kilogram basis and the direction of the force on the puck. Recall that the gravitational potential energy of a mass a height h above a reference height is given by mgh, where g is the gravitational field strength, 9.80 N/kg.

3. Consider the air hockey table of the previous question. Now assume that there <u>is</u> a friction force between the table and the puck. Prove that the presence of friction changes the overall force field from a conservative force field to a non-conservative force field. Do this by considering the work required to move a puck of mass m kg on the inclined table in a square of length x m per side.
   Hints: 1) Remember that the force of friction is given by μN, where N is the normal force between the puck and the table; 2) The total work is the sum of the work that must be done due only to the gravitational force and the work due only to the force of friction.

4. Consider the force field due to wind on the surface of a partially-treed park. The force field for the surface of the park is illustrated in Figure 2. This figure shows no force in the treed area, where there is no wind, and a non-zero force in an open area, where there is wind. Assume that the wind in the open area exerts a force on a person proportional to the mass of the person with constant F_m N/kg. (The only objects of interest are people on foot). The treed area is such that people can walk through it. Points A, B, C and D mark the corners of a rectangle.
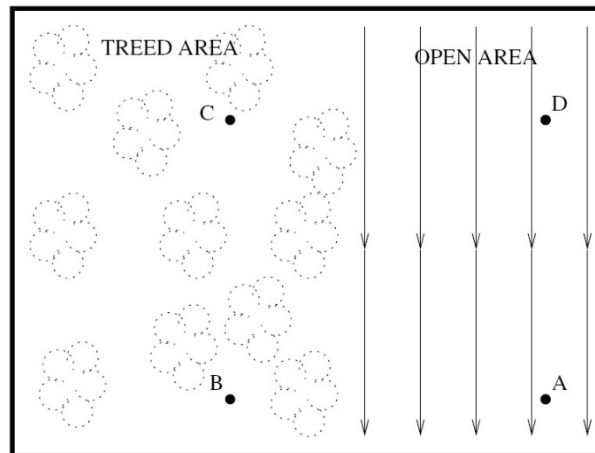
Figure 2: Force Field where force is either zero or F_m N/kg

(a) Prove that this is a non-conservative force field by considering the work required to move an object along the path A-B-C-D-A. (In a conservative force field, zero net work is required to move an object along any path that ends where it started.)

(b) Show that the concept of constant energy contours does not exist in a non-conservative force field. This can be done by first showing that the energy of an object is not changed if it is moved along path A-B-C-D in Figure 2, which means points A and D must be at the same energy levels. Then show that this is not the case (i.e. the energy <u>does</u> change) if the object is moved in a straight line from point A to D.

E0

ELECTRIC FIELDS

Created March 2006 by Professor E. Salt
Revised June 2018 by Brian Zulkoskey

PURPOSE

The purpose of this laboratory is to measure the electric field strengths and the electric potential between points in electric fields:

1. Set up by parallel planes (e.g., a parallel plate capacitor);
2. Set up by two physically separated point charges.

EQUIPMENT

The laboratory requires:

1. A 24 volt voltage source (i.e. A source of electric power that has one terminal at an electric potential of 24 V w.r.t. the other.).
2. A digital multimeter capable of measuring voltage and current. When configured as a voltmeter (a meter that measures electric potential) the multimeter should be able to measure from 0.1 V to 24 V with a precision of 1%.
3. A ruler with a millimeter scale.

REVIEW OF SOME THEORY ON FIELDS

Parallel plate capacitors

The electric field in the gap between uniformly charged parallel plates is perpendicular to the plates (the electric field is a vector denoted $\vec{E}$). This electric field has strength $E = |\vec{E}|$. The field strength is related to the voltage across the plates of a capacitor by

$$V=Ed$$

where V is the voltage across the plates (i.e., the voltage of one plate with respect to the other), E is the strength of the electric field in the gap between the plates and d is the length of the gap (i.e., the distance between the two parallel plates). Therefore

$$E = \frac{V}{d}$$

which means the electric field strength is proportional to the voltage across the plates of the capacitor.

Point charge (or uniformly charged sphere)

Due to the shell theorem, it is known that a uniformly charged sphere produces exactly the same electric field at all points outside its radius as would the same amount of charge concentrated at the center of the sphere.

Coulomb's law, in conjunction with the shell theorem, shows that the electric field lines set up by a sphere that is uniformly charged with positive charge have direction directly outward from the center of the sphere. The strength of the electric field a distance r from the center of the sphere is given by Coulomb's law, which is

$$E = \frac{kQ}{r^2}$$

where E is the strength of the electric field at any point a distance r from the center of the sphere, k is the constant $8.99 \times 10^9 \, \frac{\text{Nm}^2}{\text{C}^2}$ and Q is the amount of positive charge on the uniformly charged sphere.

The electric potential of point A with respect to point B in an electric field set up by a single sphere uniformly charged with positive charge is given by

$$V_{AB} = kQ\left[\frac{1}{r_A} - \frac{1}{r_B}\right] = -kQ\left[\frac{1}{r_B} - \frac{1}{r_A}\right],$$

where r_A is the distance from the center of the sphere to point A and r_B is the distance from the center of the sphere to point B.

Measuring Electric Fields

There is no commercially available meter that can truly measure a static electric field. The direction and strength of an electric field is determined by measuring the force exerted on a small amount of test charge. It is possible to do this for carefully designed experiments, but it is very difficult and beyond the capabilities of the equipment in our laboratories.

This laboratory is carefully designed so that an inexpensive voltmeter can be used to measure an electric field. A voltmeter is a two terminal device that measures the electric potential of one terminal with respect to the other. That is, it measures the voltage between its terminals. This laboratory experiment is designed so that a voltage measurement can be converted to a field strength measurement.
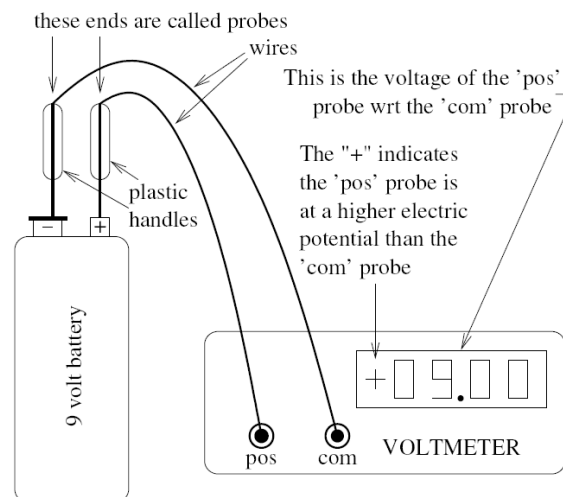
Figure 1: Connecting a voltmeter to a battery to determine the voltage across the terminals of the battery

The voltmeter terminals are extended with wires that have metallic stick-like ends called probes. Figure 1 shows how to connect a voltmeter to a standard 9 V battery to measure the voltage across the terminals of the battery.

For the voltmeter to find the electric potential of one terminal with respect to the other, charge must trickle through the meter. The electric field will force charge through the meter. Electrons will be forced into one terminal and leave through the other. The meter measures the work done by the electric field to move the charge through the meter. It converts this to a voltage reading.

Since charge is only trickling through the meter, it takes a half second or so to have enough charge pass through the meter to get an accurate measurement of the work done. The voltage is calculated and the display updated about every half second using the equation

$$V = \frac{W}{Q}$$

where V is the voltage of one 'pos' terminal w.r.t. the 'com' terminal, Q is the amount of charge that passed through the meter in the measurement interval and W is the amount of work done by the electric field on Q.

A voltmeter cannot be used to measure the electric field set up by two point charges placed in air. The reason for this is explained below. Suppose a voltmeter is used in an attempt to measure the voltage of point A w.r.t. point B in the electric field shown in Figure 2.
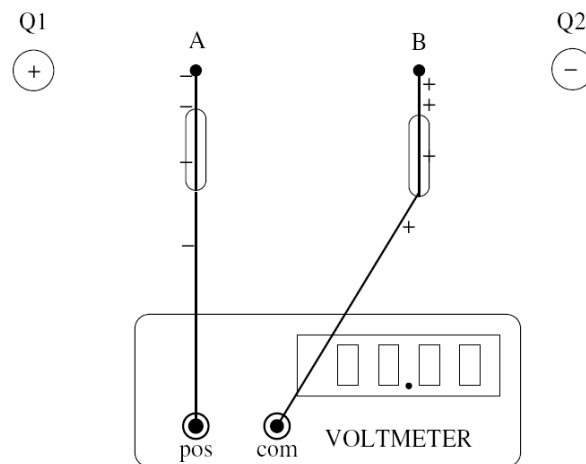
Figure 2: Problems in measuring a static electric field.

This is what would happen. As the probes were moved into position so that one is touching point A and the other is touching point B, the electric field would force charge through the meter. Since charge cannot leave the probe, any negative charge that moves through the meter must come from one probe and end up on the other. This will leave one probe positively charged and the other negatively charged. The redistribution of charge on the probes will set up an electric field that counters the original field set up by Q1 and Q2. In a very short time the two probes will become sufficiently charged to cancel the original electric field. In this steady state condition V_{AB} is zero, for if it were otherwise charge would continue to flow through the meter.

It is clear that the presence of the voltmeter greatly disturbs the electric field so a voltmeter cannot be used to measure a static electric field in such a situation.

Making the electric field measurable

An experiment can be set up so that the electric field can be measured with a voltmeter. This involves the charged particles (or plates in the case of a capacitor) that set up the electric field being linked with a sheet of conductive paper, which for this lab will be carbon-impregnated paper. Carbon-impregnated paper is conductive, but is not a good conductor like copper. The paper has considerable resistance, but charge will move through the paper when the paper is in an electric field.

If the field of interest is the field between the parallel plates then the carbon-impregnated paper must be inserted as shown in Figure 3.
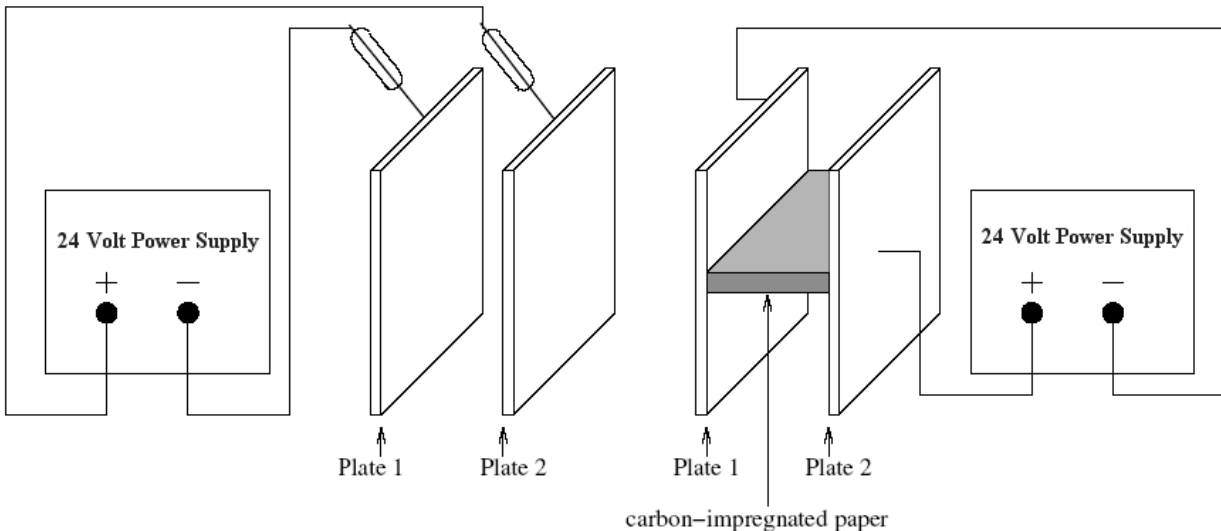
Figure 3: Parallel plates without and with carbon-impregnated paper between them.

The presence of this paper will provide a path for electrons to move from one plate to the other. The electric field set up by the charged plate will force the electrons from the negatively charged plate, through the paper, to the positively charged plate.

Normally the plates are charged by momentarily connecting a power supply as shown in Figure 3. This is done as shown on the left side of Figure 3. Probes are connected to the power supply and touched, momentarily, to the plates. As soon as the probes touch the plates, electrons flow from Plate 2 through the probe touching plate 2 down the wire to the positive terminal of the power supply. An electric field internal to the power supply (which is created by the power supply) forces the electrons from the positive terminal through the power supply to the negative terminal where they flow down the wire, through the probe touching plate 1, to plate 1. This leaves Plate 2 positively charged and Plate 1 negatively charged. When the probes are removed the excess charge in the plates remains. The amount of charge in each plate depends on the area of the plates and the distance between them and the voltage across the plates (In this case the voltage across the plates was determined by the power supply which is set at 24 V.). You are not asked to calculate the amount of charge on the plates in this laboratory, however the equation to do so was developed in class and so is given here for general interest: $Q = (\varepsilon_0 A / d)V$, where A is the area of a plate, d is the distance between the plates, V is the voltage of one plate with respect to the other (i.e., the voltage across the plates) and $\varepsilon_0 = 8.854 \times 10^{-12} \, \text{C}^2/(\text{Nm}^2)$.

Once the plates are charged the probes can be removed without changing the voltage across the plates. The plates will remain charged with an excess of electrons on one plate and an excess of protons on the other in the amount given by the equation above.

Suppose after the plates are charged and the probes removed, carbon-impregnated paper is slid between the plates as shown on the right side of Figure 3. The paper is conductive so provides a path for electrons to move from one plate to the other. The electric field set up by the charge in the plates will force electrons through the paper. The electric field moves negative charge (electrons) from the negatively charged plate to the positively charged plate. This movement of electrons reduces the amount of excess negative charge in the negatively charged plate since electrons are leaving this plate. It also reduces the amount of excess positive charge in the positively charged plate since electrons are arriving at this plate and neutralizing some of the excess positive charge. Eventually there will be no excess charge on either plate and the electric field between the plates becomes zero. The action of reducing the charge in the plates to zero is called "discharging". Plates can be discharged in a fraction of a second.

To keep an excess of charge in the plates when there is conductive paper between them, the power supply must remain permanently connected to the plates as shown on the right side of Figure 3. The power supply will force electrons from the positive plate, through the power supply, to the negative plate to replace the electrons that left through the conductive paper. The power supply keeps the voltage across the plates constant and the excess charge in each constant at $Q = (\varepsilon_0 A / d)V$.

Another advantage of using conductive paper is that it allows the experimental apparatus to be simplified. The field in the conductive paper is determined by the voltage of the plates in contact with it. Therefore the field between the plates of a parallel plate capacitor can be modeled with the apparatus shown in Figure 4.
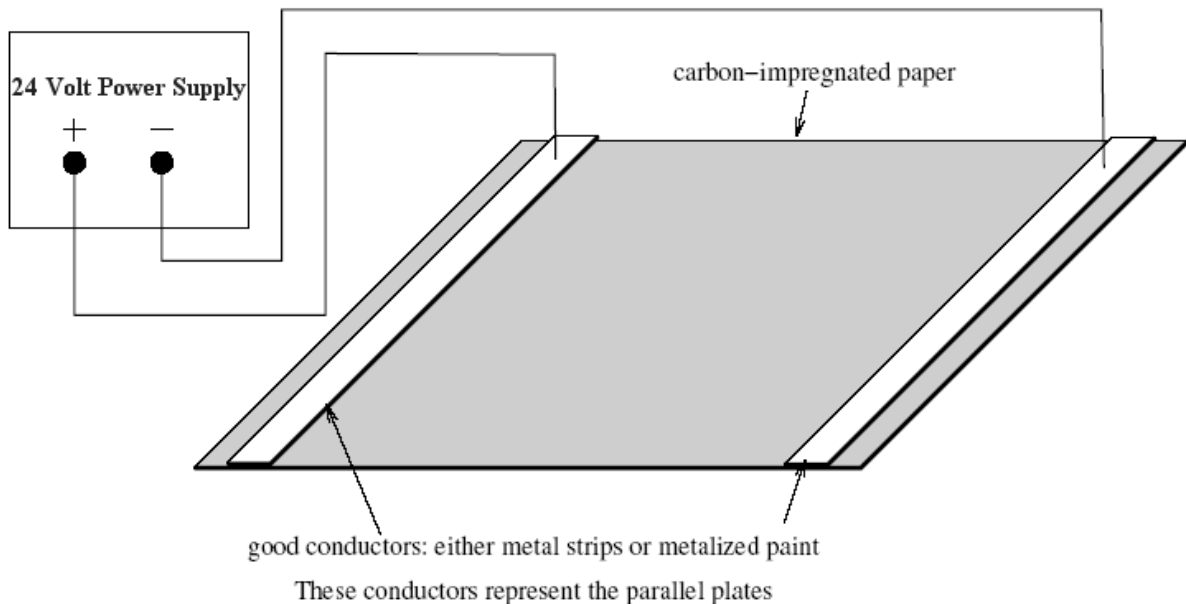
Figure 4:   A configuration where the electric field in carbon-impregnated paper is the same as a plane of the field between two parallel plates.

The electric field in the carbon-impregnated paper is the same as the electric field in a plane between the plates of a parallel plate capacitor. The orientation of the plane being modeled is perpendicular to the plates of the capacitor.

The electric field in the carbon-impregnated paper can be measured with a voltmeter. The reason for this is explained below. The electric field causes electrons to move through the paper. The electrons move along the lines of force which are the electric field lines, but the direction of movement is opposite to that of the electric field (The electric field lines have the direction of force on positive charge.). This means a map showing the flow of electrons is the same as a map showing electric field lines, except the arrows on the lines are in the opposite direction. A map showing the flow of electrons in conductive paper that is inserted between a pair of parallel plates is shown in Figure 5.
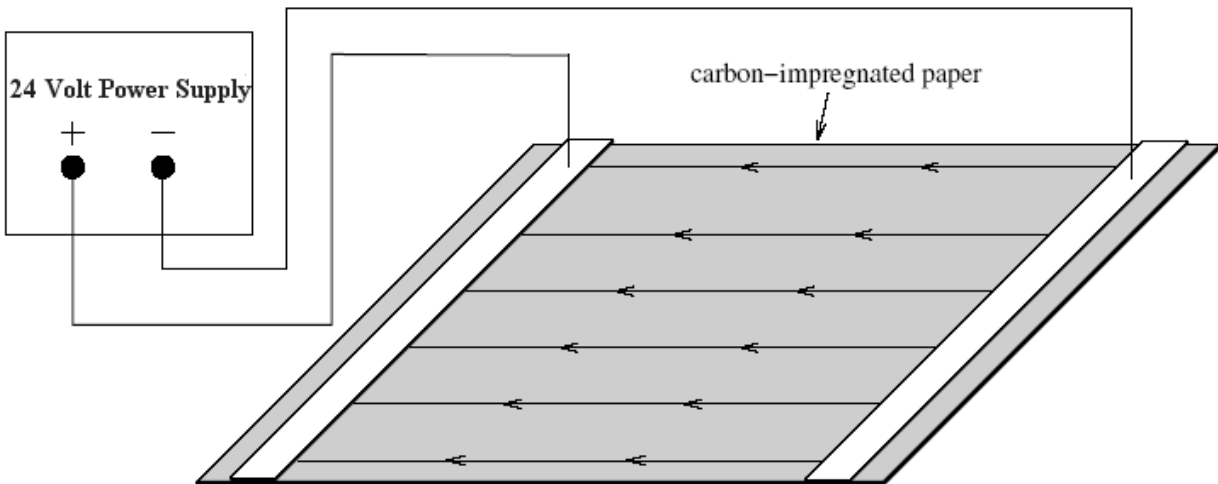
The flow of electrons is from the negatively charged to the positvely charged plate
The electrons move opposite to the direction of the electric field

Figure 5: A map showing the flow of electrons in conductive paper for an electric field generated by parallel plates.

Since electrons are flowing in the paper, a voltmeter can be used to measure the voltage between two points in the paper. Some of the electrons will flow through the voltmeter, which as explained earlier is necessary to get a voltage measurement. To illustrate the effect of electrons leaving the paper and flowing through the voltmeter, the field between two parallel plates is modeled in carbon-impregnated paper. Suppose a voltmeter is used to measure the electric potential of point A w.r.t. point B as shown in Figure 6. The presence of the voltmeter will disturb the flow of electrons and therefore change the electric field slightly. Some electrons enter the probe at point B, flow through the voltmeter, and return to the conductive paper at point A. This reduces the flow in the paper between points B and A. The dashed line in Figure 6 indicates the flow of electrons that go through the voltmeter. The electric field in the region of reduced flow is affected in two ways:

1. It reduces the electric field strength between points A and B. (The strength of flow is directly proportional to the field strength.). This effect is very slight if good carbon-impregnated paper is used.
2. It causes the flow of electrons in adjacent paths to bend toward the area where the flow has been weakened. This effect is illustrated in Figure 6. In practice, when good carbon-

impregnated paper is used, this effect is not nearly as pronounced as shown in Figure 6. It is usually very slight.
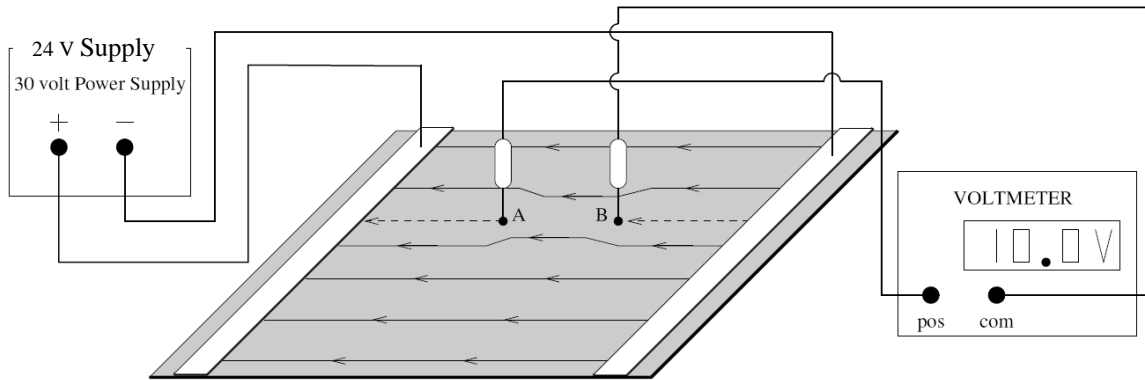
The flow of electrons is from the negatively charged to the positvely charged plate
The electrons move opposite to the direction of the electric field

Figure 6: A map showing the influence of a voltmeter on the flow of electrons in conductive paper.

The electric field strength in the direction of the line that runs through points A and B is approximately

$$E_s \approx \frac{|V_{AB}|}{\Delta x},$$

where E_s is the electric field strength in the direction of the line that runs through points A and B, V_{AB} is the voltage of point A w.r.t. point B and Δx is the distance between points A and B. If V_{AB} is positive the direction of the electric field is from A to B. (The field always point from a higher to a lower electric potential.).

EXPERIMENT

Measuring Voltage

OBJECTIVE
The objective of this part of the laboratory is to learn how to use a voltmeter and also to understand the meaning of a negative reading.

PROCEDURE
1. Measure the voltage across the terminals of a '9 V' battery. You should not expect the measured voltage to be close to 9 V. It will likely be between 4 and 6 V.
   (a) Obtain a multimeter, a 9 volt battery and two wires terminated in banana plugs.
   (b) Set the multimeter to function as a voltmeter (see experiment E1 for detailed insttructions).
   (c) On the multimeter, insert a banana plug of a black wire into the common terminal receptor and a banana plug of a red wire into the receptor used for "voltage measurement".
   (d) Use the other end of the wires, which are terminated in banana plugs, as probes. Touch the probe from the common terminal receptor of the voltmeter to the negative (larger) terminal of the battery. Touch the other probe to the positive (smaller) terminal of the battery. The voltmeter will then read the electric potential of the

positive (smaller) terminal of the battery with respect to the voltage of the negative (larger) terminal. Draw the circuit in your laboratory notebook showing how the multimeter is set, what it reads and how it is connected to the battery. Draw the battery in a way that clearly distinguishes the positive (smaller) and negative (larger) terminals.

(e)   Reverse the probes so that the probe from the common terminal receptor of the voltmeter is touching the positive (smaller) terminal of the battery and the other probe from the voltmeter is touching the negative (larger) terminal of the battery. What does the voltmeter read? Explain why the voltmeter reads a negative electric potential.

Check the offset calibration error of the voltmeter as per instructions below. The "offset" error is the value the voltmeter reads when the voltage of one probe with respect to the other is 0 V. Obviously, if the voltmeter is properly calibrated the offset will be zero.

(a)   Remove the voltmeter probes from the battery, but leave them connected to the voltmeter.

(b)   Touch the probes together forcing the voltage between the probes to be zero.

(c)   Draw the circuit in your laboratory notebook showing how the multimeter is set and show the two probes connected together.

(d)   What is the offset error of the voltmeter you are using?

Set up a power supply so that there is 24 V across its terminals by following the steps below:

(a)   Obtain a power supply.

(b)   Set the power supply to 24 V.

(c)   Use the voltmeter to measure the electric potential of the 'pos' terminal w.r.t. the 'neg' terminal. Draw the circuit in your laboratory notebook showing how the multimeter is set, how it is connected to the power supply, and what it reads.

Charting the electric field between parallel plates

OBJECTIVE

In this section of the laboratory the task is to chart the electric field between uniformly charged parallel plates.

Charting an electric field involves plotting both the electric field lines and the equipotential lines. In this experiment the equipotential lines are found by measurements and sketched in the laboratory notebook. The electric field lines are then found by freehand drawing lines perpendicular to the equipotential lines.

PROCEDURE

1. Obtain the following equipment: A 24 V power supply; a digital multimeter; two wires (with banana plugs ends); an apparatus for holding carbon-impregnated paper; carbon-impregnated paper; two metal strips. Throughout the experiment, be careful not to make any marks on the black carbon-impregnated paper.

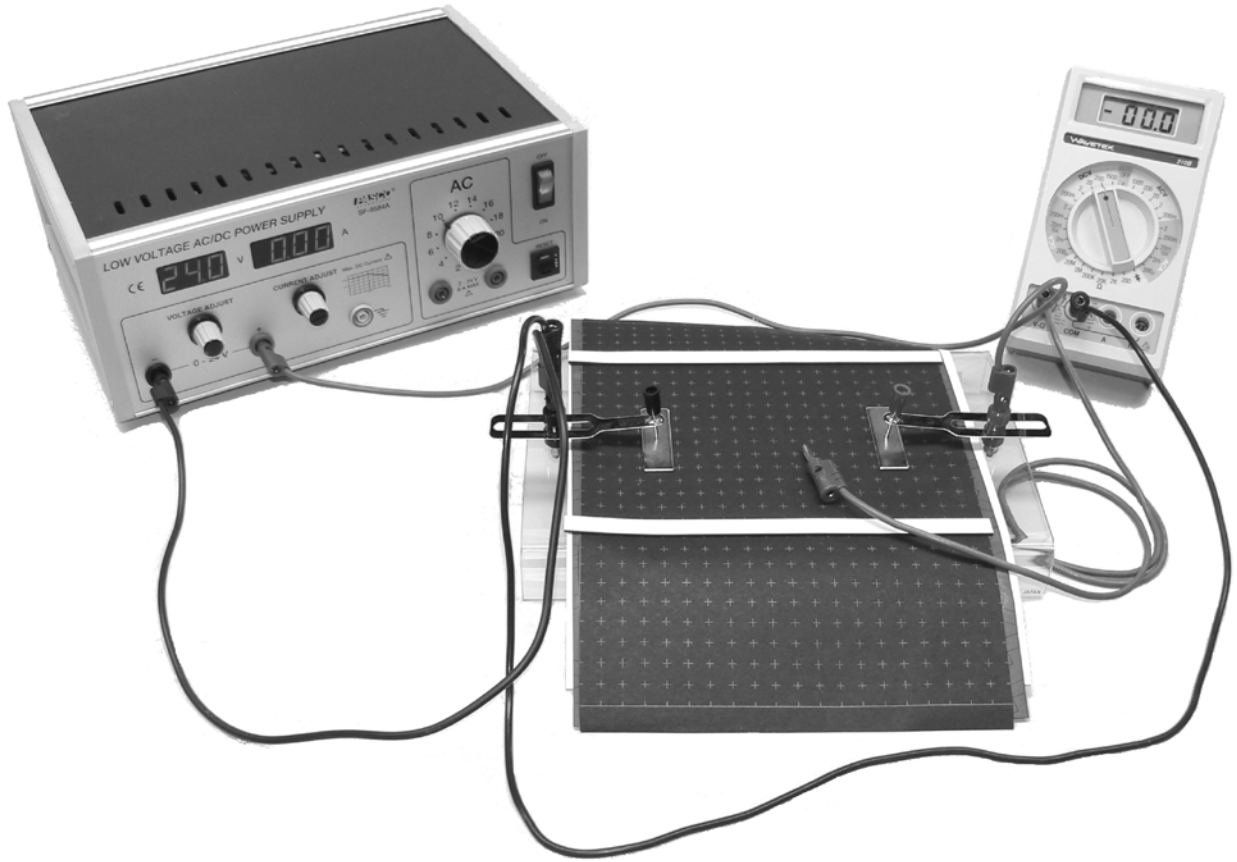2. Connect the apparatus as shown in Figure 7.

Figure 7: A configuration to be used to measure the equipotential lines in the field between two parallel plates.

3. In your laboratory notebook, draw the plane of the carbon-impregnated paper showing the position of the metal strips. As this lab progresses this drawing will become a map of the electric field, therefore care must be taken to draw to scale. A suitable scale is to have one square on the carbon-impregnated paper correspond to a two-by-two square (i.e. 4 grid squares) in your notebook.

4. Keeping the common terminal of the voltmeter connected to the metal strip that is connected to the negative terminal of the power supply, probe the carbon-impregnated paper with the positive probe of the voltmeter to find a point along the centre line between the metal strips that is 12.0 V with respect to the metal strip that is connected to the negative terminal of the power supply. Carefully record the location of the point in your laboratory notebook.

5. Find four more 12.0 V points so that a 12.0 V equipotential line can be drawn. Plot the positions of the 12.0 V points in your laboratory notebook and then using these points draw the 12.0 V equipotential line.

6. Repeat the procedure to find four more equipotential lines. Find the 4.0 V, 8.0 V, 16.0 V and 20.0 V equipotential lines.

7. Draw an electric field line on your equipotential map. Draw the line that intersects the middle of the metal strip that is connected to the negative terminal of the power supply.

8. Using the approximation

$$|\vec{E}| \approx \frac{\Delta V}{\Delta x},$$

where Δx is the distance between equipotential lines, calculate the approximate value for the electric field strength at a point along the field line (the field line drawn in step 7.) that is halfway between the 8.0 V and 12.0 V equipotential lines. Use a ruler to measure the distance Δx. Remember to scale your measurement if the scale of your field map is not 1:1.

Charting the electric field between point charges

OBJECTIVE

The objective of this section of the laboratory is to chart equipotential lines and electric field lines between two charged spheres and also to determine the amount of excess charge on each sphere.

PROCEDURE

1. Carbon-impregnated paper is used to model the electric field in a plane that passes through the center of two spheres. The spheres can be modeled as disks (pennies) on the carbon-impregnated-paper. This is illustrated in Figure 9.

   Set up a circuit as shown in Figure 9.
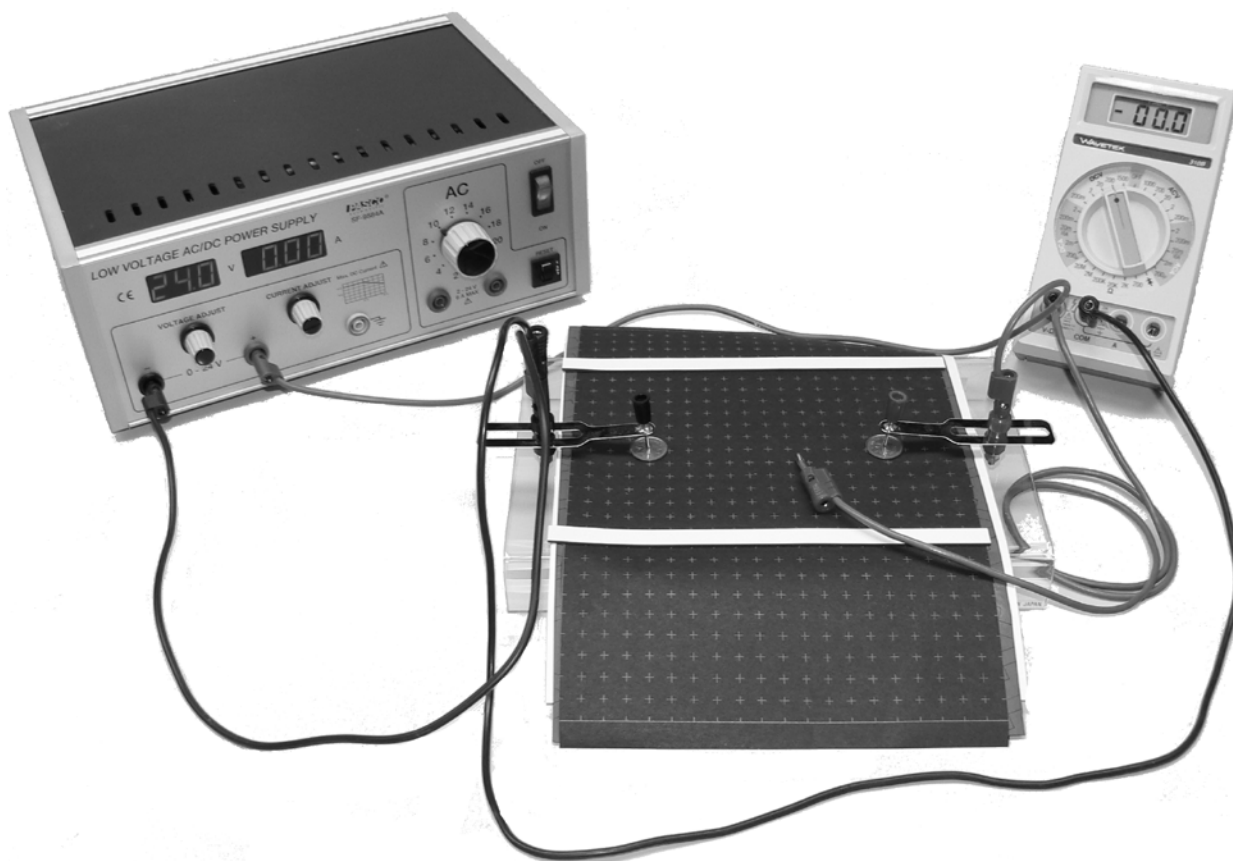
Figure 9: Test apparatus for measuring the equipotential and electric field lines set up by two charged spheres.

2. Find the 4.0 V, 8.0 V, 12.0 V, 16.0 V, and 20.0 V equipotential lines using the same procedure as was used to find the equipotential lines between the parallel plates. Carefully plot the points corresponding to these equipotential lines to scale in your laboratory notebook.

3. Draw five electric field lines on the plot of equipotential lines. First draw the electric field line that leaves the positively charged disk (modeling the positively charged sphere) in the direction toward the center of the other disk (modeling the negatively charged sphere). Then draw the electric field lines that leave the positively charged disk at angles of about ±20 degrees and ±40 degrees measured from the line joining the centers of the disks. This is illustrated in Figure 10. **Remember that electric field lines are always perpendicular to equipotential lines**.
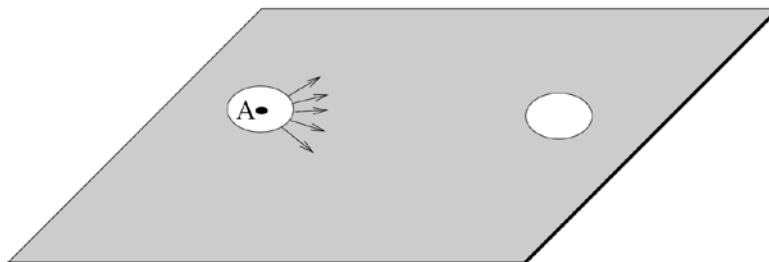
Figure 10: Illustration of the beginnings of the electric field lines to be drawn.

4. Calculate the electric field strength at Point B, where B is the intersection of a straight line that runs through the centers of the disks (spheres) and the 12.0 V equipotential line. Use the equation

$$| \vec{E} | \approx \frac{\Delta V}{\Delta x},$$

taking $\Delta V = 16.0$ V – 8.0 V and $\Delta x = $ the distance between the 8.0 V and 16.0 V equipotential lines along the line that runs through the centers of the disks.

5. Calculate the excess charge on each disk using the following logic that results in an equation for Q.

The excess charge on a uniformly charged sphere sets up the same field (at points external to the sphere) as the same amount of charge concentrated at a point at the center of the sphere. The electric field strength due to a point charge at a distance r from the point charge is

$$E = 8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \times \frac{|Q|}{r^2}, \tag{5}$$

where Q is the amount of charge.

The electric field at point B is the sum of the electric fields due to the two point charges. The electric field caused by each point charge is in the same direction so the resultant electric field will have the same direction. The resultant electric field will have a magnitude equal to the sum of the magnitudes of the fields due to each point charge.

The strength of the electric field (i.e. the magnitude) due to each charge will be very nearly the same for the following reasons:

1. The excess charge on both disks (i.e. Q) will be the same in magnitude but opposite in sign. The reason is that the negative charge that leaves one disk ends up on the other. Therefore the magnitude of Q used in equation (5) will be the same for both point charges.

2. Point B is on the 12.0 V equipotential line, which should be very nearly halfway between the two disks. Thus the distance from each point charge to the point where the electric field is evaluated (i.e. point B) is virtually the same. This means the value of r used in equation (5) will be the same for both point charges.

Therefore the electric field measured at point B is given by

$$E_{\mathrm{B}} = 2 \times 8.99 \times 10^9 \, \frac{\mathrm{Nm}^2}{\mathrm{C}^2} \times \frac{|Q|}{r^2}.$$

and the amount of excess charge on one of the disks is given by

$$|Q| = \frac{E_{\mathrm{B}} r^2}{2} \times \frac{1}{8.99 \times 10^9} \frac{\mathrm{C}^2}{\mathrm{Nm}^2}$$

where r is the distance between point B and the center of a disk (i.e., half the distance between the centers of the two disks) and E_{B} is the value of electric field calculated in step 4.

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E1

MEASUREMENTS OF VOLTAGE AND CURRENT, AND THE VOLTAGE-CURRENT RELATIONSHIP FOR SEVERAL TYPES OF RESISTANCE

OBJECTS

♦ To measure the voltage of a variable voltage supply, using a digital multimeter;
♦ To use the power supply in an electric circuit in order to determine the voltage-current relationship for a linear load;
♦ To convert a fixed voltage into a variable supply by means of a rheostat;
♦ To measure the voltage-current relationship for a nonlinear load (a light bulb);
♦ To determine the resistivity of the rheostat wire.

EQUIPMENT

In this experiment you will use a power supply, a digital multimeter (DMM), an analogue ammeter, and a rheostat.

POWER SUPPLY

An electrical circuit is a simple example of an engineering 'system'. In this type of system, there is an energy supply (the power supply or battery) and an energy absorber or load (which in this lab is a simple resistor), and an energy transmission path linking the supply and absorber (in this case the wires AC and DB which provide a closed path or loop for the current which transmits energy from supply to load).
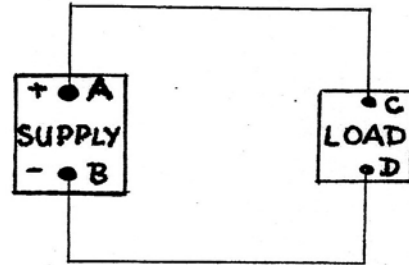
This system is analogous to the hydraulic brake system on an automobile. The supply is the energy obtained from your foot pressure on the brake, the absorber or load is the set of brake pads, and the transmission path is the fluid that transmits the pressure from the brake pedal to the brake pads.

You will use the power supply from E0 in this lab:

The Low Voltage Power Supply that was used in experiment E0 can produce up to 24 volts of electrical potential difference; i.e. it can perform up to 24 Joules of work on each Coulomb of charge that moves around the circuit. It can move this charge around the circuit at a rate of up to 10 Coulombs per second; i.e. the maximum current is 10 A. The current flows out of the RED (+) terminal, through the circuit, and back into the BLACK (–) terminal.
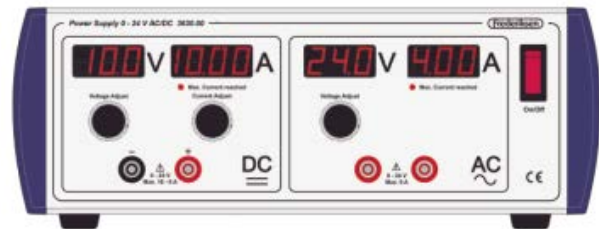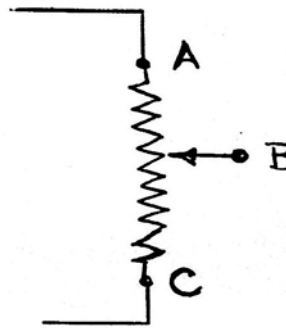
RHEOSTAT

A rheostat is a 3-terminal resistance device constructed from a cylindrical coil of wire.
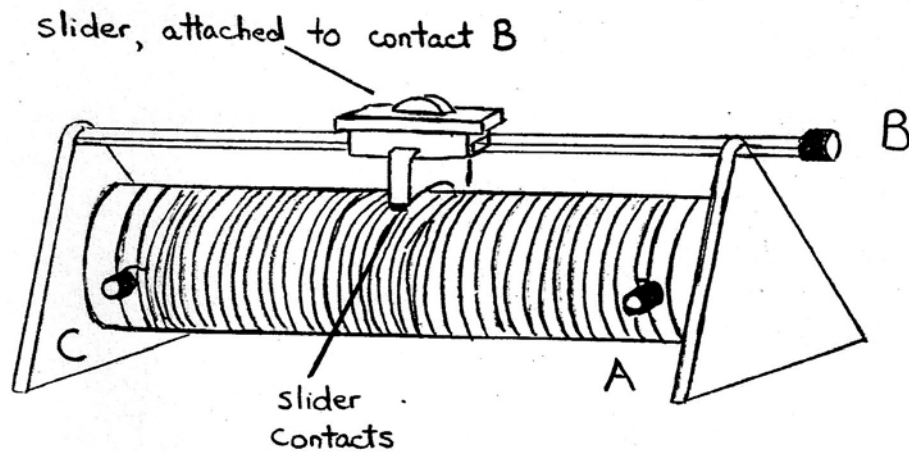
The full rated resistance (common lab values are 40 Ω, 85 Ω, and 170 Ω) is across the terminals A and C. The slider, which can be moved along the coil between A and C, has contacts that touch the coil between A and C. These contacts are attached to point B.

Electrically, the rheostat can be drawn simply as shown, with the arrow indicating that point B can be moved from one end (A) to the other end (C) of the rheostat.

RHEOSTAT AS A VARIABLE VOLTAGE CONTROLLER

In this mode of operation, the whole supply voltage V_s is applied across the fixed rheostat terminals A and C. The load is attached between the slider B and the fixed end C. Thus if the slider contact B is moved from end C to end A the load voltage $V_{LOAD} = V_{BC}$ goes from 0 to V_s respectively.
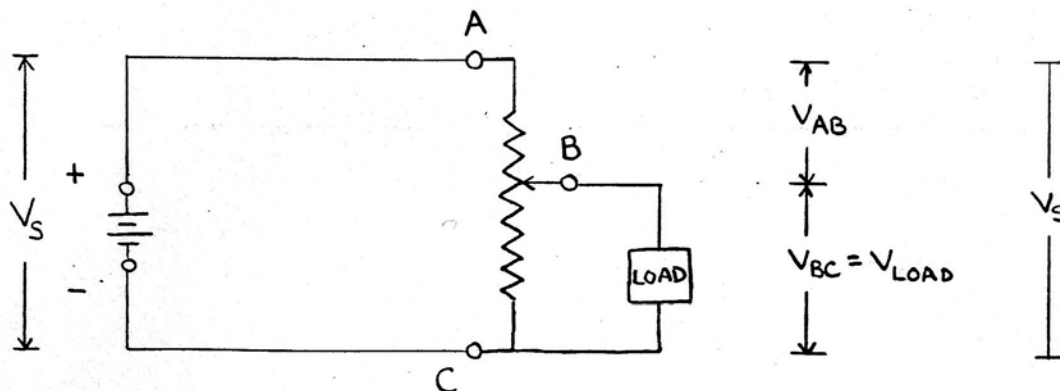
Thus, by using the rheostat, we can control the voltage applied across the load.

DIGITAL MULTIMETER (DMM)
You have already used a digital multimeter to measure DC voltage in experiment E0. A typical DMM has the ability to measure DC and AC voltages, DC and AC currents, and resistance. Additional features may include the ability to measure frequency, capacitance, temperature, etc.

Also, typically, there is one terminal that is common to all connections, a second terminal for connecting the meter to measure voltage or resistance, and a different second terminal for connecting the meter to measure current.

Making the proper connections is very important, since it prevents damage to the instrument and determines what measurement you will be making. To measure voltage or resistance, insert a black wire in the middle 'COM' terminal. Normally this is at ground (0 voltage) or at the low-voltage point when measuring voltage. Next insert a red wire into the 'VΩ' terminal. This enables you to measure either voltage or resistance, depending on the setting of the central dial.

To measure current, the 'mA μA' or '10 A' terminal and the 'COM' terminal are used. The meter is connected so that it is **in series** in the part of the circuit through which the current is to be measured, and so that the current is flowing into the 'A' terminal and out of the 'COM' terminal.

Meterman 33XR DMM
<u>DC Voltage</u>: To measure DC voltage connect the positive lead to the terminal labeled V and the other lead to the COM terminal. The DCV ($\overline{\overline{V}}$) section of the central dial has five divisions (400m, 4, 40, 400 and 1000) marked with the MAXIMUM voltage that can be handled without overloading the meter circuitry. The smaller numbered divisions have greater accuracy. To avoid overloading the circuitry, first turn the dial to the 1000 scale. If the voltage (or the maximum voltage expected) is less than the next-lower division's maximum (400 volts) you can safely rotate the dial clockwise to the next division. You can repeat this procedure until you reach the division with the greatest accuracy that does NOT overload the meter. If you reach the 400m scale the meter will read in milliVolts.

<u>AC Voltage</u>: To measure AC voltage, connect the leads to the V and COM inputs. Turn the dial counter-clockwise into the ACV ($\tilde{V}$) range. Follow a similar procedure to that described under DC Voltage to avoid overloading the meter.

<u>DC Current</u>: To measure DC current connect the positive circuit lead to the 10A input terminal and the other lead to the COM input. Turn the dial to the 40m/10A position in the DCA ($\overline{\overline{A}}$) range. The meter will now read in amps to a maximum of 10 amps.

If the meter reads less than 300 mA, and you are sure that the maximum expected current will ALWAYS be under 300 mA, then you can safely obtain a more accurate reading by turning the meter off and reconnecting the positive lead to the A input. (NOTE: you will blow a fuse, and possibly damage the meter, if the current ever exceeds 300 mA.) The dial can now be turned to the 300 mA setting in the DCA range. The meter will now read in milliamps. If the current is less than 40 mA, turn the dial to the 40m/10A setting for greater accuracy. You can continue to switch to finer settings provided you do NOT exceed the MAXIMUM current indicated on the dial. The finest setting reads in microamps to a maximum of 400 microamps.

AC Current: Follow the same procedure as described under DC Current except that the dial is rotated into the ACA ($\tilde{\text{A}}$) range.

Resistance: To measure the resistance of a circuit element, first either disconnect the electrical connections of that element from any other part of the circuit or ensure that there is NO voltage supplied to the circuit element. Then connect a lead from one end of the element into the ohm (Ω) input and a lead from the other end into the COM input. Rotate the dial into the ohm (Ω) range starting with the coarse scale (4M) where resistance will be read in Mega-ohms (MΩ). You can rotate the dial to finer settings providing the resistance does NOT exceed the MAXIMUM RESISTANCE indicated. If you use the 40k setting the resistance will read in kilo-ohms (kΩ). If you use the 400 division, the resistance will read in ohms (Ω) to a maximum of only 400 ohms.

Amprobe 37XR-A DMM

DC Voltage: To measure DC voltage connect the positive lead to the terminal labeled V Ω and the other lead to the COM terminal. Turn the dial clockwise to the $\overset{\overline{\overline{...}}}{\text{V}}$ setting, which allows measurement of DC voltages from milliVolts to 1000 V (auto-ranging).

AC Voltage: To measure AC voltage, connect the leads to the V Ω and COM inputs. Turn the dial clockwise to the ACV ($\tilde{\text{V}}$) setting, which allows measurement of AC voltages from 10s of milliVolts to 750 V (auto-ranging). Frequency range is 45 Hz to 2 kHz.

DC Current: To measure DC current connect the positive circuit lead to the 10A input terminal and the other lead to the COM input. Turn the dial to the 10A position in the Ammeter range. The meter will now read in amps to a maximum of 10 amps.

If the meter reads less than 400 mA, and you are sure that the maximum expected current will ALWAYS be under 400 mA, then you can safely obtain a more accurate reading by turning the meter off and reconnecting the positive lead to the mA μA input. (NOTE: you will blow a fuse, and possibly damage the meter, if the current ever exceeds 400 mA.) The dial can now be turned to the mA setting in the Ammeter range. The meter will now read in milliamps. If the current is less than 1 mA, turn the dial to the μA setting for greater accuracy.

AC Current: Follow the same procedure as described under DC Current except that you must depress the yellow button to change the meter from measuring DC Current to measuring AC Current.

Resistance: To measure the resistance of a circuit element, first either disconnect the electrical connections of that element from any other part of the circuit or ensure that there is NO voltage supplied to the circuit element. Then connect a lead from one end of the element into the ohm (V Ω) input and a lead from the other end into the COM input. Rotate the dial to the Ω (ohm) setting. The meter can read up to 40 MΩ with a maximum precision of 0.1 Ω (for resistances less than 1000 Ω).

ANALOGUE AMMETER
When using an analogue ammeter, you must be careful of the direction of current flow. The ammeter has a red banana plug socket labelled '+' and a black socket labelled '–'. Current must flow INTO the red '+' terminal, and OUT OF the black '–' terminal.

Often there are several black terminals, one for each current range that the ammeter measures. (On some meters, there may be one black terminal and several red terminals, one for each current range.

The proper current range to use is the one which results in the largest possible deflection of the ammeter needle without causing the needle to go off-scale (which will damage the meter). That is, the more sensitive scales (with <u>lower</u> maximum currents) have a <u>lower</u> current per scale division ratio and so a given current corresponds to a <u>larger</u> number of divisions than would be the case for a less sensitive scale.
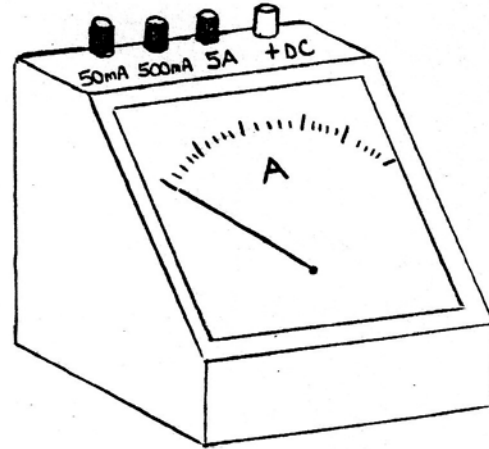
For example, suppose that a meter has 50 scale divisions. On a 5 A scale, each division corresponds to 1/10 A, or 100 mA, and on a 500 mA scale, each division corresponds to 10 mA. Therefore, a 200 mA current corresponds to 2 divisions on the 5 A scale and 20 divisions on the 500 mA scale. Since the experimental uncertainty is ±1 division for both scales, the best scale to use is the 500 mA one, since it gives a measurement of 200 ± 10 mA (20 div ± 1 div) whereas the 5 A scale yields 200 ± 100 mA (2 div ± 1 div).
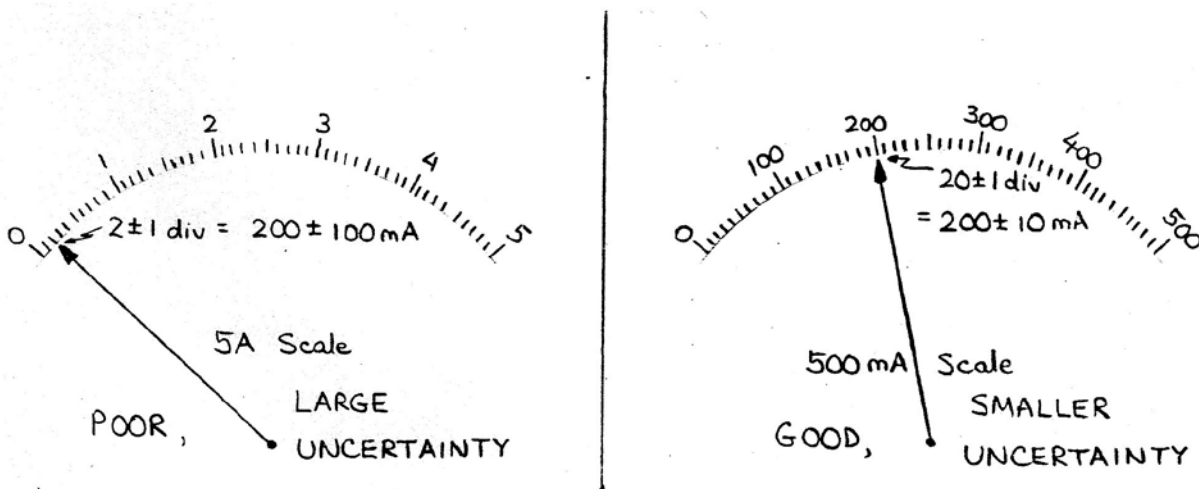
When measuring an unknown current, initially set the ammeter on the least sensitive scale (the one that can handle the largest maximum current). Then, if the meter deflection is too small, the next, more sensitive scale can be chosen. This method of starting with the least sensitive scale and then trying more sensitive scales (e.g. 5 A, then 500 mA, then 50 mA) protects the ammeter, since it is possible that the unknown current exceeds the maximum currents on some of the more sensitive scales, and the meter would be damaged if these scales were used.

PRINCIPLES OF ELECTRIC CIRCUIT MEASUREMENT

All the experiments that you perform in this class involve measurement of voltage, current and/or resistance.

MEASUREMENT OF VOLTAGE AND RESISTANCE

Both voltage and resistance are measured <u>across</u> a circuit element. That is, the measuring instrument is connected in <u>parallel</u> with the circuit element whose voltage drop or resistance is to be measured.

As an example, consider the following circuit:
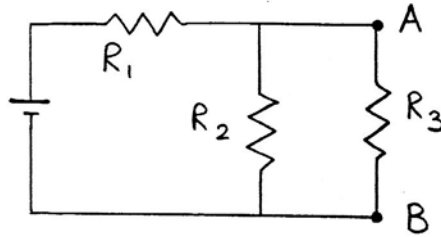
To measure the voltage drop across resistor R_1, the measuring instrument is connected in the following way:
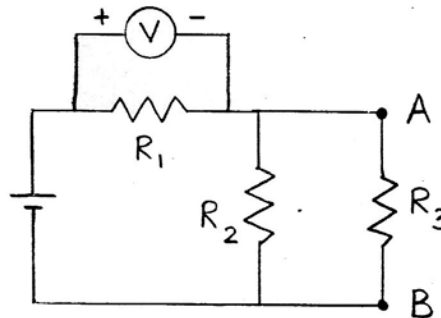
When measuring resistance, the circuit must be disconnected from all voltage sources (power supply, battery, etc.). Also, if the circuit element whose resistance is to be measured is connected to other circuit elements, then the meter reading may or may not equal the resistance of the desired circuit element, depending on how the other circuit elements are connected.

To measure the resistance of R_1 , the measuring instrument is connected in the following way:
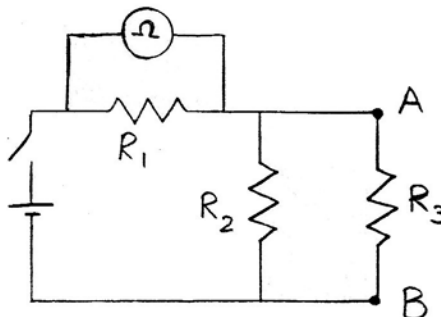
Note that the switch from the power supply must be opened (the power supply must be turned **OFF**) when measuring resistance.

If the resistance meter is connected at points A and B, as shown below, the measured resistance is NOT R_3, but instead is the equivalent resistance of the parallel combination of R_2 and R_3.
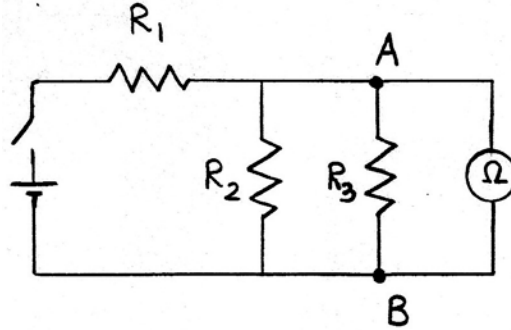

To obtain a value for the resistance of just R_3, R_3 would have to be physically disconnected from the other resistors:
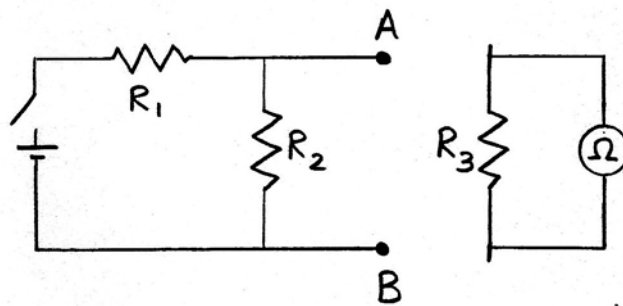
MEASUREMENT OF CURRENT
Current is measured with an ammeter. To measure current, the current must flow <u>through</u> the ammeter, so the ammeter is connected in <u>series</u> in the portion of the circuit where current is to be measured.

To measure the current flowing through R_1, in the circuit example used previously, the ammeter is connected in the following way:
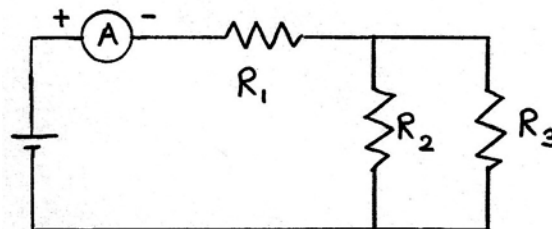
Note that the circuit must be 'broken' or 'opened' so that the ammeter can be inserted into the branch of the circuit containing R_1.

THEORY
OHM'S LAW
When a voltage *V* is applied across a circuit or component the resulting current *I* that flows
through the circuit or component is related to the applied voltage by a property of the circuit or
component called its resistance *R*.

The resistance of a circuit or component is defined as the ratio of the applied voltage to the
resulting current.

$$R = \frac{V}{I} \tag{1}$$

When the resistance is constant this relationship between voltage, current and resistance is called
Ohm's Law. Depending on what is to be determined, Ohm's Law can be written as:

$$V = IR \; ; \quad R = \frac{V}{I} \; ; \quad I = \frac{V}{R} \tag{2}$$

Note that a plot of voltage versus current for a resistance obeying Ohm's Law is linear with slope
equal to the resistance. A plot of voltage versus current is called the voltage-current
characteristic.

NONLINEAR RESISTANCE
A nonlinear resistance is one whose resistance is NOT constant. The voltage-current
characteristic of a nonlinear resistance will be a curve rather than a straight line.

The DYNAMIC resistance of a nonlinear resistance at a specific value of voltage or current is
the slope of the tangent to the voltage-current characteristic at that value of voltage or current.
That is, the dynamic resistance is the change in voltage divided by the change in current where
the (small) voltage and current intervals are taken around the value at which the dynamic
resistance is to be determined.

RESISTIVITY AND RESISTANCE
Resistivity, ρ, is the property of a material that describes its opposition to the flow of electric
charge (opposition to electric current).

Resistance, *R*, depends not only on the type of material, but also on the shape of the piece of
material.

The resistance of an object of uniform composition and uniform cross-sectional area is given by:

$$R = \frac{\rho \ell}{A} \tag{3}$$

where ρ = resistivity of the material of the object, ℓ = length, and *A* = cross-sectional area

In this experiment, the resistivity of the wire of a rheostat will be determined. The resistance R will be measured directly with an ohmmeter. To determine ℓ and A the following measurements will be made:

> n = number of turns of wire in 10 cm (0.10 m) of rheostat coil
> L = length of rheostat coil
> C = circumference of rheostat coil

Then $A = \dfrac{\pi D^2}{4}$ where D = wire diameter = $\dfrac{0.10 \text{ m}}{n}$

and ℓ = length of wire on rheostat = $\dfrac{CnL}{0.10 \text{ m}}$

can be used in equation (3) to obtain:

$$\rho = \frac{\pi}{4}(0.10\text{m})^3 Rn^{-3}C^{-1}L^{-1} \qquad (4)$$

EXPERIMENT
Part A:
1. (a) Set the current knob to maximum clockwise (max CW). The current knob acts as a current regulator. It can be set to limit the current from exceeding a value that might be harmful to your circuit. By setting the current knob at 'MAX' you allow the power supply's maximum current to be delivered, if the load requires it. Measure the voltage delivered by the power supply when its voltage control knob is set as low as possible (max CCW (counter-clockwise)), at mid-range, and at max CW.

   (b) After completing the above measurements, return the voltage control knob to its lowest setting (max CCW).

2. Measure the resistance of resistor AB on the resistor circuit board, by using the DMM as an ohmmeter.

3. (a) You will now determine the relationship between current and voltage for resistor AB by connecting the power supply, the analogue ammeter, and the DMM voltmeter to resistor AB.
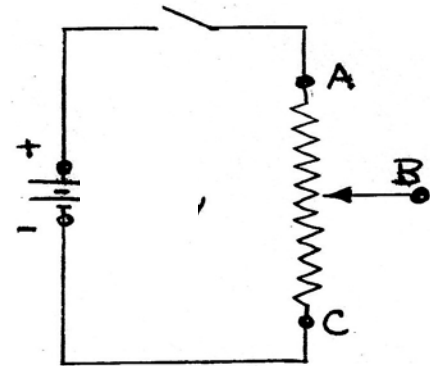
   Draw the diagram of this circuit and <u>have your diagram and your connections checked by the instructor before proceeding</u>.

   Measure current and voltage for resistor AB as the voltage is changed from 3 to 12 volts in 3 volt increments. Adjust the voltage of the power supply so that the voltage across the resistor is as close as possible to the desired value – **and then record the actual value of the resistor voltage and record the resistor current**. *Ignore the voltage shown on the power supply's built-in voltage display.*
   (b) Return the voltage control knob to its lowest setting (max CCW).

Part B:
4. (a) Connect the fixed ends of A and C of the rheostat
to the power supply. The voltage V_{AC} will equal
the power supply voltage. Set the power supply
voltage to its maximum. The voltage between the
sliding contact B and the fixed end C can be varied
by moving the slider. Connect the DMM (in
voltmeter mode) across B and C. Measure the
voltage V_{BC} as B is moved from end C to end A of
the rheostat. Record the voltage reading V_{BC} when
the slider is:

    i)   ¼ the distance from C to A.
    ii)   ½ the distance from C to A.
    iii) ¾ the distance from C to A.
    iv) at A.

(b) Return the voltage control knob to its lowest setting (max CCW).

Part C:
5. Nonlinear Voltage-Current Characteristic
The voltage-current characteristic of an incandescent light bulb will now be measured.

Referring to the circuit used previously in Part A, step 3, draw the diagram of the circuit that
you will use in this part of the procedure.

***Have your circuit diagram and your connections checked by the instructor before
proceeding.***

Measure current and voltage for the light bulb as the current varies from **100 mA to 450 mA
in 50 mA steps (*be sure to use the correct ammeter scale*)**. As before, keep the current knob
at its maximum setting and obtain the various desired current values by changing the applied
voltage.

Part D:
6. The resistivity of the rheostat wire can be calculated by measuring the total resistance of the
rheostat and determining the total length of wire wound on the rheostat and the cross-
sectional area of this wire.

Measure R, the total resistance of the rheostat. (Use the DMM as a resistance meter and
connect its leads to the A and C terminals of the rheostat.)

Loop a piece of string around the rheostat coil and measure the rheostat's circumference, C.
This is the length of wire used in one turn of the coil.

Count the number of coil turns, n, in a 10 cm (0.10 m) length of the coil.

Measure L, the length of the rheostat coil.

ANALYSIS
Part A:
   Plot a graph of voltage versus current for resistor AB. If the graph is linear, calculate its slope. What is the significance of the slope value? Compare the slope of the graph to the resistance value measured in step 2.

Part B:
    Comment on the relationship between slider position and output voltage (V_{BC}).

Part C:
   For each pair of current and voltage values, calculate the light bulb resistance ($R = V/I$).

   Plot a graph of resistance versus current for the light bulb. Discuss the shape of this graph.

   Plot a graph of voltage versus current for the light bulb. Does the light bulb obey Ohm's Law?

   From the graph of voltage versus current, determine the dynamic resistance of the light bulb at 200 mA and at 400 mA by determining the slopes of the tangents to the plotted curve at these two currents. How do these values compare with those calculated from Ohm's Law? (In electronics, many devices are nonlinear and are operated over a small range of current centred on an operating value I_o . Under these conditions, the dynamic resistance at that operating value I_o is very important.)

Part D:
   Using your measurements in equation (4), calculate the resistivity of the rheostat wire.

   Compare your calculated resistivity with the table of values listed in your textbook. If you do not have your textbook with you, compare your value with the table of resistivities posted in the lab. Your value should agree with (or be 'close' to) the value of one of the listed materials. Which one is it?

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E2

DC MEASUREMENTS OF SIMPLE RESISTANCE NETWORKS

OBJECT
♦ To measure and calculate the equivalent resistance of various resistor networks;
♦ To observe and account for the loading effect of a voltmeter.

THEORY
Resistors are marked with coloured bands which are interpreted according to a colour code. The code is:

0 = BLACK	5 = GREEN
1 = BROWN	6 = BLUE
2 = RED	7 = VIOLET
3 = ORANGE	8 = GREY
4 = YELLOW	9 = WHITE
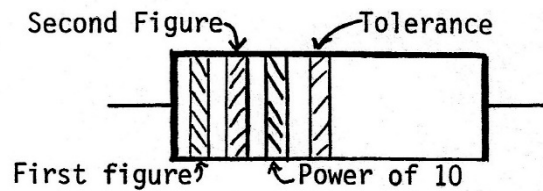
The first two bands give the two significant figures of the resistor value in terms of the above code, the third band gives the power of 10 which multiplies the first two figures. For example, a Yellow-Violet-Green resistor would be 47×10^5 Ω (4.7 MΩ), and a Blue-Green-Orange resistor would be 65×10^3 Ω (65 kΩ). The fourth band gives the resistor tolerance (accuracy), according to the code:

no band at all	± 20%
silver band	± 10%
gold band	± 5%

Thus a Red-Violet-Blue-Gold resistor is 27×10^6 Ω ± 5%

RESISTOR NETWORKS
When a number of resistors are connected together in a circuit they behave electrically as if they were a single resistor. The value of this single EQUIVALENT resistance can be calculated using the laws of circuit analysis, namely Ohm's Law and Kirchhoff's Laws.

Ohm's Law: The resistance R between two points in a circuit equals the voltage drop V across these two points divided by the resulting current I between these two points.
$$R = V/I$$

Kirchhoff's Voltage (Loop) Law
(KVL): In any closed loop the algebraic sum of the voltage drops and rises equals zero.
i.e. The change in voltage in going around a closed loop is zero.

Kirchhoff's Current (Node) Law
(KCL): At any node the algebraic sum of the currents entering and leaving the node equals zero. i.e. The total current entering a node equals the total current leaving the node.

In many instances the arrangement of resistors in a network consists of combinations of resistors in series and parallel. In these instances simple rules can be used to calculate the equivalent resistance.

Resistors in Series: The equivalent resistance of any number of resistors connected in series is the sum of the individual resistor values.

Consider a voltage source E connected to 3 resistors in series as shown.
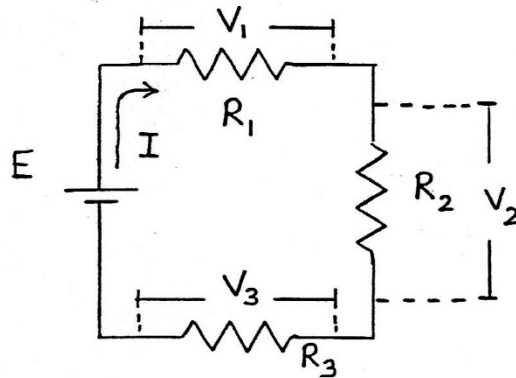
Let the current be I. From KCL the current is the same through each of the 3 resistors.

From KVL: $E = V_1 + V_2 + V_3$

From Ohm's Law: $E = IR_1 + IR_2 + IR_3$

$$E = I(R_1 + R_2 + R_3)$$

$$E/I = R_1 + R_2 + R_3$$

But from Ohm's Law E/I is the equivalent resistance, R_{SER}, of the network connected across the terminals of the voltage source.

Therefore, $R_{SER} = R_1 + R_2 + R_3 = \Sigma R_i$

Resistors in Parallel: The equivalent resistance of any number of resistors connected in parallel is the inverse of the sum of the inverses of the individual resistor values.

Consider a voltage source E connected to three resistors in parallel as shown:
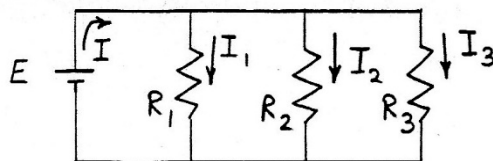
From KVL the voltage drop across each of the resistors is E. Let I be the total current drawn from the voltage source, and let I_i be the current through R_i.

From KCL: $I = I_1 + I_2 + I_3$

From Ohm's Law: $I = E/R_1 + E/R_2 + E/R_3$

$$I = E(1/R_1 + 1/R_2 + 1/R_3)$$

$$I/E = 1/R_1 + 1/R_2 + 1/R_3$$

$$E/I = (1/R_1 + 1/R_2 + 1/R_3)^{-1}$$

Therefore, $R_{PAR} = (1/R_1 + 1/R_2 + 1/R_3)^{-1}$

or

$$\frac{1}{R_{PAR}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \sum \frac{1}{R_i}$$

In some instances the equivalent resistance cannot be calculated by using the series and parallel rules. The equivalent resistance is then found by direct application of Ohm's and Kirchhoff's Laws:
1. A voltage source of arbitrary value E is imagined to be connected across the terminals between which the resistance is to be calculated.

2. Arbitrary current directions are assigned to each resistor.

3. Enough voltage loops are drawn around the circuit so that each resistor is contained in at least one voltage loop.

4. KVL and KCL are applied to the loops and nodes to obtain simultaneous equations in the unknown currents.

5. The simultaneous equations are solved and the equivalent resistance of the network is E/I where I is the total current drawn by the network.

EFFECT OF A VOLTMETER
An ideal measuring device is one that does not affect the value to be measured. Since voltmeters are connected in parallel across the two points whose potential difference is to be measured, an ideal voltmeter is one that draws no current. That is, the ideal voltmeter has infinite resistance.

A REAL voltmeter has a finite resistance, so it can be represented as an ideal voltmeter (infinite resistance) with a finite shunt resistance R_v.
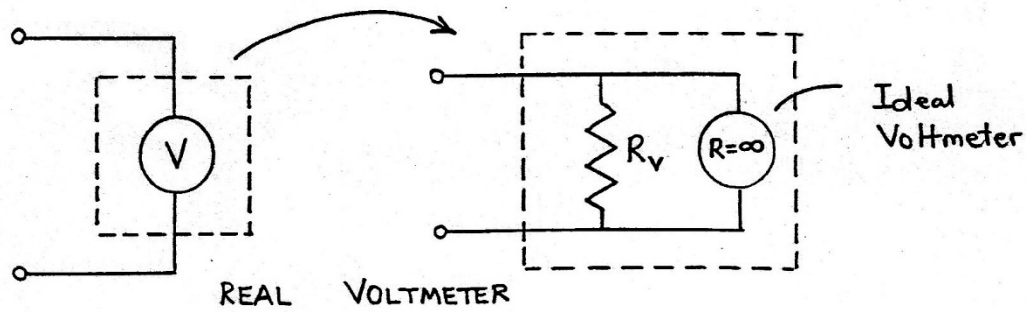
REAL VOLTMETER

Consider the effect when a real voltmeter is used to measure the voltage across one of two resistors in series that are connected to a voltage source E.
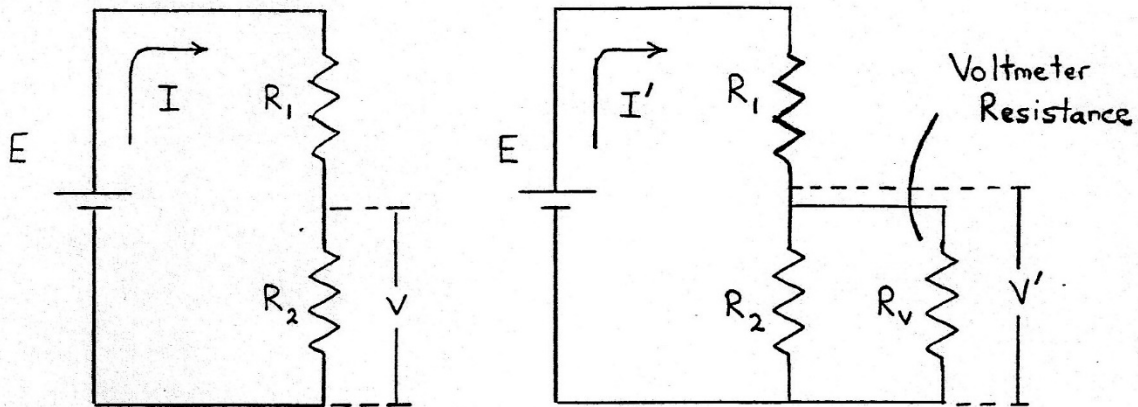
BEFORE the voltmeter is connected, the total equivalent resistance of the circuit is $R_1 + R_2$. The current flowing in each of the resistors is

$$I = \frac{E}{R_1 + R_2}$$

and the voltage drop V across R_2 is

$$V = IR_2 = \frac{E}{R_1 + R_2} R_2 = \frac{E}{\left(\dfrac{R_1}{R_2} + 1\right)}$$

If $R_1 = R_2 \ (= R)$ then

$$V = \frac{E}{R_1 + R_2} R_2 = \frac{E}{R + R} R = \frac{E}{2R} R = \frac{E}{2} = \tfrac{1}{2}E$$

When the voltmeter is connected across the second resistor, R_2, to <u>measure</u> its voltage drop, the effect is to add a resistance R_v in parallel with the resistor R_2. If R_2' is the equivalent resistance of this parallel combination then

$$R_2' = \left(\frac{1}{R_2} + \frac{1}{R_v}\right)^{-1} = R_2\left(\frac{R_v}{R_v + R_2}\right) = R_2\left(\frac{1}{1 + \frac{R_2}{R_v}}\right) < R_2$$

The new total equivalent resistance, R'', of the circuit is

$$R'' = R_1 + R_2'$$

The new current flowing in the circuit is

$$I' = E/R''$$

and the new voltage drop across R_2 is

$$V' = I'R_2' = \frac{E}{R''}R_2' = \frac{E}{R_1 + R_2'}R_2' = \frac{E}{\left(\frac{R_1}{R_2'} + 1\right)}$$

and $\qquad\qquad\qquad V' < V$ since $R_2' < R_2$.

That is, the effect of a voltmeter is to give a reading that is less than the voltage that was present before the voltmeter was connected.

The larger the value of R_v, the closer R_2' is to R_2 and the smaller the effect of the voltmeter. Thus good voltmeters have very large resistances in order to minimize their effect.

EXPERIMENT

1. Tabulate the colour bands and nominal resistance value of each of the resistors on the board.
    e.g.

Circuit Pts	Colour Code	Resistance
AB	orange-orange-brown-gold	$33 \times 10^1 \ \Omega \pm 5\%$
BC	...	...

2. Direct Measurement of R_{eq}:
    Construct the resistor network as shown by joining A with D and
    C with F.
    Using the DMM as an ohmmeter, measure the equivalent
    resistance between points 1 and 2 by connecting the DMM at A
    and C.

    Ohm's Law Determination of R_{eq}:
    You will now use the DC power supply, the analogue ammeter
    on the 50 mA scale, and the DMM as a voltmeter. Connect these
    components to the resistor network so that the current drawn by,
    and voltage drop across, the network are measured. Draw the
    circuit diagram.

    *Have your circuit diagram and your connections checked by an
   instructor, before turning on the power supply.*

    *After your connections have been checked and approved, turn
    on the power supply and slowly turn the power supply voltage
    to its maximum value (~24 V). Record the current drawn by,
    and voltage drop across, the resistor network.*

3. Leaving A joined with D, and C with F, use another lead to
    connect points B and E. Measure the equivalent resistance of the
   circuit by using the DMM as an ohmmeter.

    As before, now set up the circuit to use the DC power supply,
    analogue ammeter, and digital voltmeter to measure the current
    drawn by, and voltage drop across, the network.

    *Have your circuit diagram and your connections checked by an
    instructor, before turning on the power supply.*

    *After your connections have been checked and approved, turn
    on the power supply and slowly turn the power supply voltage to
    its maximum value (~24 V). Record the current drawn by, and
    voltage drop across, the resistor network.*
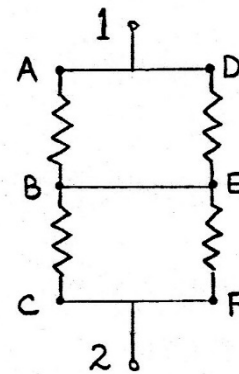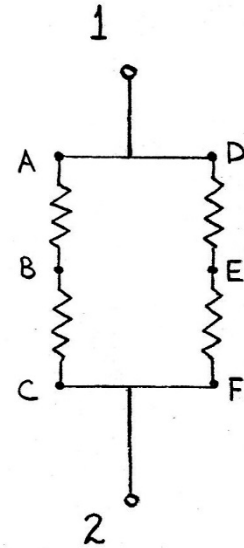
4. Consider the network of resistors connected between points K, L, M, N, O, and P. Using the DMM in ohmmeter mode, measure the equivalent resistance between K and L, between M and N, between M and P, and between K and P.
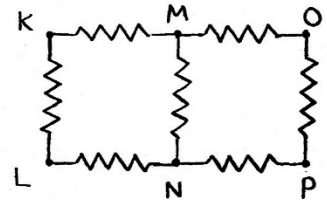
5. Effect of a Voltmeter on a Circuit
   a) Set the DC power supply to 10.0 V.
   b) Connect the power supply across terminals G and J of the resistor board.
   c) Measure the voltage drop across points HJ.

ANALYSIS
1. <u>Ohm's Law Determination of R_{eq}:</u>
   For the circuit studied in step 2. of the experiment (A connected to D, C connected to F), calculate the equivalent resistance between points 1 and 2 by using Ohm's Law.

   <u>Series/Parallel Calculation of R_{eq}:</u>
   Calculate the equivalent resistance between points 1 and 2 by applying the series and parallel resistor rules to the colour code values recorded for the resistors in the network.

   Compare the three values (directly-measured, Ohm's Law, Series/Parallel) obtained for the equivalent resistance between 1 and 2.

2. For the circuit studied in step 3. of the experiment (A connected to D, C connected to F, B connected to E), determine the equivalent resistance of this circuit by the methods used in 1. (directly-measured, Ohm's Law, Series/Parallel). Compare the three values (directly-measured, Ohm's Law, Series/Parallel) obtained for the equivalent resistance.

3. Consider the network of resistors connected between points K, L, M, N, O, and P.
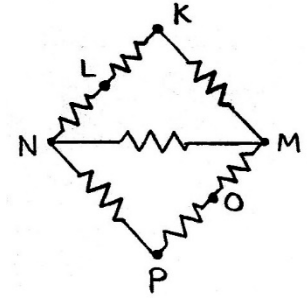

   a) Calculate the expected equivalent resistance between points K and L by applying the series/parallel rules to the colour code values. Re-draw the circuit diagram to clearly show which resistors are in parallel and which are in series. Compare the measured and calculated equivalent resistances.
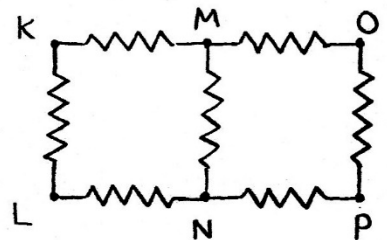
   b) Calculate the resistance between M and N and compare with the measured value. Note that all seven resistors are still part of the circuit and that the re-drawn circuit will be different from the one for a).

   c) Calculate the resistance between M and P and compare with the measured value. Again, note that all seven resistors are still part of the circuit and that the re-drawn circuit will be different from the ones for a) and b).

d) *OPTIONAL: (worth 1 bonus mark if solved correctly, by the specified method; worth ½ bonus mark if equations are set up using the specified method, but answer is incorrect)*

Calculate the equivalent resistance between K and P using loop analysis. Compare the measured and calculated values. Notice that the circuit can be redrawn as shown, which facilitates setting up the loop equations.

4. Effect of a Voltmeter on a Circuit
   a) Calculate the expected voltage across HJ by using the colour code values for the two resistors, the supply voltage (10.0 V) and assuming the voltmeter to be ideal (infinite resistance). i.e. Calculate the voltage across HJ when the voltmeter is not connected.

   b) Now calculate the voltage across HJ using a value of 10.0 MΩ for the voltmeter resistance.

   c) Compare the voltages calculated in a) and b) with the measured voltage across HJ. Discuss the significance of your results.

E4

THE MAXIMUM POWER TRANSFER THEOREM

OBJECT
♦ To measure the power transferred from a source to a load, and the efficiency of this power
transfer, as functions of load resistance;

THEORY
A. MAXIMUM POWER TRANSFER THEOREM
Consider a voltage source E with internal resistance R_i , connected to a load resistance R_L , as
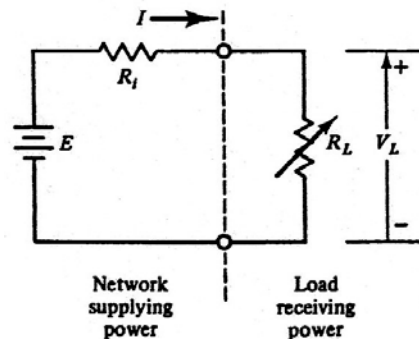shown in the following figure:

Figure 1

The current for any value of load resistance is

$$I = \frac{E}{R_i + R_L}$$

The power delivered or transferred to the load is

$$P_L = I^2 R_L = \frac{E^2 R_L}{\left(R_i + R_L\right)^2}$$

It can be shown that the power transferred to the load for a load resistance of $R_L = R_i / n$ is equal
to the power transferred to the load for a load resistance of $R_L = nR_i$ (n any number). The power
transferred to the load changes rapidly with a change in load resistance when $R_L < R_i$, and the
power transferred to the load changes much more slowly with load resistance when $R_L > R_i$.
Because of this dependence on load resistance of the rate of change of power transferred to the
load, it is often convenient to plot power transferred as a function of the logarithm of load
resistance.

By differentiating the expression for P_L with respect to R_L and setting the result to zero, it is
found that P_L is a maximum when $R_L = R_i$. This is the maximum power transfer theorem:
maximum power is transferred from a source when the load resistance equals the internal

resistance of the source. Note that under the condition of maximum power transfer, the load voltage V_L is one-half of the open-circuit source voltage E.

The efficiency of the power transfer to the load, η, is defined as the power transferred to the load, P_L, divided by the total power developed by the source, P_D $(= EI)$.

$$\eta = \frac{P_L}{P_D} = \frac{V_L I}{EI} = \frac{V_L}{E} = \frac{R_L}{R_i + R_L}$$

Note that maximum efficiency does not coincide with maximum power transfer. Under maximum power transfer conditions, the efficiency is only 50%. The efficiency increases as load resistance is increased, reaching the ideal efficiency of 1 when load resistance is infinite (and power transferred is 0). To increase the efficiency without sacrificing too much load power, a load resistance of two to three times the internal resistance of the source results in an appreciable reduction in wasted power (as heat in the source) for only a small reduction in power transferred to the load. A load resistance less than the internal resistance of the source not only results in a reduction in power transferred to the load but also causes a very high dissipation within the source. This condition of operation is termed overload and must be avoided.

In practice, the trade-off between maximum power transfer and high efficiency must be reconciled by considering the purpose for which a system is being designed. In electronics and communications systems, it is usually important to obtain maximum power from low power sources, thus the source and load resistances are matched for maximum power at the expense of efficiency. Electric power companies, however, try to keep their losses low by operating at high efficiency.

EXPERIMENT

NOTE: Throughout this experiment, rather than performing error calculations you can assume an error of 5% in all <u>calculated</u> values. Remember to still record experimental uncertainties in all measured values.

A. EXAMINATION OF POWER TRANSFER TO A LOAD

The following components will be used to examine the power transferred to a load and the efficiency of this power transfer as a function of load resistance: a DC power supply, an ammeter, and resistor AB from the resistance circuit board comprise the "network supplying power" and a decade resistance box comprises the "load receiving power". The internal resistance of the source, R_i, is taken to be the sum of the ammeter resistance and R_{AB}.

1. Set the ammeter on the 300 mA DC setting (33XR) or the mA setting (37XR-A). Using the other digital multimeter (DMM) in ohmmeter mode, measure the resistance of the series combination of the ammeter and resistor AB from the resistance circuit board. This is the internal resistance R_i.

2. Set the DC power supply to 15.0 V. This is the source voltage E.
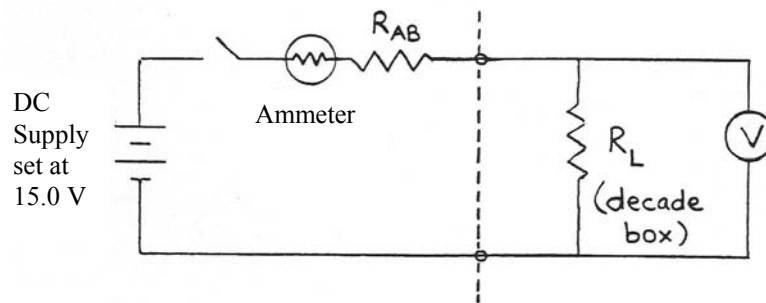
3. Connect the circuit shown in Figure 3.

Figure 3

Have your circuit connections checked by an instructor before proceeding.

4. Measure the load voltage and current for load resistances (decade box settings) of 20, 40, 60, 80, 100, 150, 200, 250, 300, 350, 400, 500, 600, 800, 1000, 1500, 2000, 2500, 5000, and 7500 Ω.

5. Calculate the natural logarithm of the load resistance, the power transferred to the load $(P_L = V_L I)$, the power dissipated in the source $(P_i = I^2 R_i)$, the total power developed by the source $(P_D = EI)$, and the efficiency $(\eta = P_L/P_D)$. Tabulate your results in the following format:

R_i = _____ ± _____ kΩ
E = _____ ± _____ V

Load Resistance $(\Omega \pm 1\%)$	$\ln(R_L)$ (± 0.01)	Current I $(\pm 0.1$ mA$)$	Load Voltage V_L $(\pm 0.01$ V$)$	Load Power P_L (mW) $\pm 5\%$	Lost Power P_i (mW) $\pm 5\%$	Total Power P_D (mW) $\pm 5\%$	Efficiency η $\pm 5\%$
20	3.00						
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.	.	.	.	.	.	.	.

6. Plot the power transferred to the load, P_L, and the efficiency, η, versus load resistance, R_L for values of R_L from 20 to 1000 ohms. Both plots may be made on a single graph page by using two different vertical scales, one for the load power and one for the efficiency.

7. Plot the power transferred to the load, P_L, and the efficiency, η, versus the logarithm of the load resistance, $\ln(R_L)$, for values of R_L from 20 to 7500 ohms. Both plots may be made on a single graph page by using two different vertical scales, one for the load power and one for the efficiency.

8. From each graph, determine the load resistance at which maximum power is transferred to the load. Estimate the experimental uncertainty in these values. Compare with the value of R_i measured in 1. Do your results agree with the prediction of the maximum power transfer theorem? From each graph, what is the efficiency at maximum power transfer? Estimate the uncertainty in each value. Compare with the theoretical efficiency at maximum power transfer.

9. From the graph in 7 determine the power transferred to the load at a load resistance of $R_i/3$ and at a load resistance of $3R_i$. Estimate the uncertainty in these values. Do they agree with each other within your estimated uncertainty? Should they? Explain.

10. Discuss the observed dependence on load resistance of power transferred to the load and the observed dependence on load resistance of efficiency, in terms of the equations discussed in the theory.