MEASUREMENT OF HALF-LIFE

The radioactivity of a short-lived radionuclide is measured as a function of time and the half-life of the substance is determined.

Theory:

The Geiger-Muller Counter

The counter consists of a cylindrical chamber (tube) with a wire stretched along its longitudinal axis and insulated from its walls. The chamber walls act as the cathode, and a positive voltage is applied to the wire, making it the anode. The cylinder is filled with a low pressure gas mixture of argon and ethyl alcohol.

When an ionizing particle passes through the counter, it collides with gas molecules in the chamber. The collision liberates electrons, which are attracted to the centre wire (anode). As the electrons travel towards the anode, they collide with other gas atoms, liberating more electrons. These collisions change the tube voltage. An event pulse is produced at the anode as a result of the electrons striking the anode and the change in tube voltage. This pulse is amplified and counted.

The local depletion of electrons creates an ion sheath around the anode, rendering the counter inoperative until the ion sheath has been dissipated. The time required for this dissipation to occur is called the DEAD TIME of the counter and is typically of the order of hundreds of microseconds.

A number of factors must be taken into account when analysing counting experiments. Three of these factors are background radiation, counter losses (efficiency), and statistics.

Background radiation or more commonly, background is the term applied to events that are registered even when no target radiation source is present. This is due to contamination in the lab from other experiments, building materials, the soil, and cosmic rays. To obtain the rate due to the radiation source alone, the background counting rate must be subtracted from the total rate.

As already mentioned, during the development of a discharge, the voltage at the anode is lowered to such an extent by the ion sheath that a second particle passing through at that time will not be counted. This "dead" time is denoted $\tau_D$. If $m'$ is the observed counting rate, the actual rate is given approximately by

$$m = \frac{m'}{1 - m' \tau_D}$$

(1)

Since $\tau_D$ is small, of the order of 100 microseconds, this correction is usually only applied for counting rates higher than 100/sec (i.e. when $m' \tau_D$ is significant compared to 1, say $m' \tau_D > 0.01$).
Statistics

The count rate obtained in any time interval will fluctuate from the average counting rate over a long period of time according to the laws of probability. Counting experiments involving radioactive decay, or gamma radiation absorption obey a Poisson statistical distribution. For a single measurement of $n$ counts the standard deviation is equal to $\sqrt{n}$, and the result of a measurement is quoted as $n \pm \sqrt{n}$. (In some cases, the result of a measurement is quoted as $n \pm 2\sqrt{n}$ since an uncertainty of $2\sigma$ corresponds to 95% confidence in the result). To obtain a relative uncertainty of 1%, for example, one must have $\sqrt{n}/n = 0.01$, or $n = 10,000$ counts. One must always count long enough to obtain sufficient counts in order to have the desired accuracy.

If $N$ amount of runs are made under the same conditions, the results are given by $\bar{n} \pm \sqrt{\bar{n}/N}$ where $\bar{n}$ is the mean. That is, the standard deviation of the mean of $N$ runs, $\bar{n}$, is reduced by a factor of $\sqrt{N}$. The probable error for a single measurement is $0.67\sqrt{\bar{n}}$. This means that if a series of measurements was made under the same conditions and the deviation from the mean $(n - \bar{n})$ was calculated, half of the deviations would be greater than $0.67\sqrt{\bar{n}}$ and half would be less.

Radioactive Decay (Determination of Half-life)

Radioactive decay is a statistical phenomenon: every type of radioactive nucleus has a certain probability of decaying in a given time span, but it is impossible to predict to absolute certainty exactly which, or even how many, nuclei will actually decay in a particular sample during that time span. By analogy to coin flips, you expect a coin to come up heads half the time. But if you flip a coin 10 times, you can’t be sure that there will be exactly 5 heads and 5 tails, and you certainly can’t be sure that the first, third, fourth, sixth, and ninth coin flips will be heads.

Let $\lambda$ be the constant probability per unit time for the decay of each nucleus of a given substance. Since $\lambda$ is the probability per unit time, $\lambda \Delta t$ is the probability that any nucleus will decay in a short time interval $\Delta t$. If a sample contains $N(t)$ undecayed nuclei, the number $\Delta N$ that decay in a time $\Delta t$ is the product of the number of nuclei $N(t)$ and the probability $\lambda \Delta t$ that each will decay in $\Delta t$.

i.e.  
\[ \Delta N = - N(t) \lambda \Delta t \] (2)

The minus sign is required since $N(t)$ is decreasing with time ($\Delta N$ is intrinsically negative since it is the loss of $N$). Note that because $N(t)$ continuously decreases with time, equation (2) cannot be solved directly because the variation of $N$ with $t$ is unknown (in fact, determining $N(t)$ is the purpose of this discussion). This problem can be overcome by taking the infinitesimal limit of the time interval, i.e. $\Delta t \to dt$. Then $\Delta N \to dN$ and $N(t)$ can be replaced by $N$, the number of nuclei at the beginning of the infinitesimal time interval, since $dN$ is infinitesimally small compared to $N$ during the interval $dt$.

\[ dN = - N \lambda \, dt \] (3)

and the number of undecayed nuclei remaining as a function of time can be found by integration.
The activity, $A$, of a radioactive sample is given by

$$A = -\frac{dN}{dt} = \lambda N$$  \hspace{1cm} (8)

By defining $A_o = \lambda N_o$,

$$A = A_o e^{-\lambda t}$$  \hspace{1cm} (9)

That is, the count rate of radiation emitted from a radioactive source decays exponentially with time.

The half-life is the time interval over which the activity of the source falls to one-half its value at the beginning of the time interval ($A = \frac{1}{2} A_o$). Let $t_{1/2}$ be the half-life. Then

$$\frac{1}{2} A_o = A_o e^{-\lambda t_{1/2}}$$  \hspace{1cm} (10)

where $A_t$ is the activity at the beginning of the time interval, and

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$  \hspace{1cm} (11)

**Applications:**

A common knowledge application of half-life measurements is radiocarbon dating. Interactions between cosmic rays and $^{14}$N nuclei in the atmosphere produce $^{14}$C. This carbon is chemically identical to $^{12}$C, and so it forms organic compounds inside of living matter. The ratio between atmospheric $^{14}$C and $^{12}$C has been fairly constant throughout history and matches the ratio found within living beings. When a plant or animal dies, it stops exchanging carbon with its surroundings. At this point, the $^{14}$C starts decaying back to $^{14}$N. By measuring the activity in an organic sample (such as a wooden tool or a piece of cloth), the number of half-lives it has undergone can be determined. This can then be converted to number of years since the specimen stopped exchanging carbon with the atmosphere, which usually corresponds with the age of the sample (i.e. assuming the wooden tool was constructed at roughly the same time as the tree was
Scientists have carefully measured and catalogued the $^{14}$C activity of many samples of known age. This is important because events in the past (e.g. nuclear weapons testing in the 1950s) has changed the atmospheric levels of $^{14}$C in the atmosphere, so a straightforward half-life calculation would not be accurate. Radiocarbon dating is accurate for samples roughly 600 – 60,000 years old. Samples newer than 600 years have not undergone enough reduction in activity to be distinguishable from modern samples ($^{14}$C has a half-life of 5730 years). Samples older than 60,000 years have undergone so many half-lives that the amount of radioactive material is negligible.

**Apparatus:**

![Figure 2](image)

The apparatus consists of a PASCO SN-7927A Geiger-Muller counter tube/Power Supply, a PASCO 550 interface, and a computer.

The PASCO 550 supplies voltage to the G-M tube, counts the pulses from the anode, and provides the interface between the G-M tube and the PC. The high voltage for the G-M tube is pre-set by its integrated power supply.

The radiation source used in this experiment is a liquid $^{137}$Ba source produced by elution in a ‘generator’. Care should be taken not to spill the $^{137}$Ba solution.

**$^{137}$Ba Source Generator**

The radioisotope generator is a commercial unit. It contains a long-lived ‘parent’ isotope, $^{137}$Cs (half-life of 30 years) and its short-lived ‘daughter’ isotope, $^{137}$Ba$^{m}$. Both isotopes are absorbed into a small ion exchange resin column. In order to measure the half-life of the $^{137}$Ba$^{m}$, it must first be separated from the $^{137}$Cs. As it happens, both the $^{137}$Cs and the resin are insoluble in a solution of hydrochloric acid and sodium chloride called the ‘eluent.’ However, the daughter radionuclide
Half-life.5

$^{137}\text{Ba}^m$ resulting from the beta decay of $^{137}\text{Cs}$ is soluble in the eluent. Thus when the eluent is passed through the column of the generator, the resulting liquid contains dissolved $^{137}\text{Ba}^m$ but not $^{137}\text{Cs}$. $^{137}\text{Ba}^m$ decays to its ground state by the emission of a 0.662 MeV gamma ray which is detected and counted by the Geiger-Muller tube/PASCO 550. The radioactive decay scheme for $^{137}\text{Cs}$ is shown in Figure 1.

\[ \begin{align*}
\text{\begin{tabular}{c}
$^{137}\text{Cs}$
\end{tabular}} & \xrightarrow{\beta^-} \begin{tabular}{c}
$^{137}\text{Ba}^m$
\end{tabular} \\
\xrightarrow{\gamma} & \begin{tabular}{c}
$0 \text{ MeV}$
\end{tabular}
\end{align*} \]

**Figure 1: Decay of $^{137}\text{Cs}$ to $^{137}\text{Ba}$ (through $^{137}\text{Ba}^m$)**

**Procedure and Experiment:**

1. Log in to the computer (if requested, the password is brianlab).
2. Turn on the PASCO 550.
3. Double-click the desktop icon for PASCO Capstone. The measurement parameters will be set using the Capstone software, and the collected data will be displayed within Capstone.
4. Click Table and Graph in Capstone.
5. Click Hardware Setup.
   a) In the graphic of the interface, click 1
   b) Scroll down and click Geiger Counter
   c) Click Hardware Setup to close the Hardware Setup window
6. Data Table Configuration
   a) Click <Select Measurement> in the left column of the data table.
   b) Click Geiger Counts (counts/sample)
   c) Click <Select Measurement> in the right column
   d) Click Time (s)
7. Graph Configuration
   a) Click <Select Measurement> on the $y$-axis
   b) Click Geiger Counts (counts/sample)
   c) Click <Select Measurement> on the $x$-axis
8. At the bottom middle of the window, select the desired count frequency (5.00 min for background, 30.00 s for the half-life measurements)

9. Before doing the half-life measurement, the background radiation must be measured. Be sure that all radiation sources are removed from the vicinity of the G-M tube.

10. Background radiation is counted as follows:
   a) Set the count frequency to 5.00 min.
   b) Click the round red 'Record' button at the bottom left of the screen. (The button will become a square red 'Stop' button when data is being collected.)
   c) Click ‘Stop’ after one count interval (5.00 min) is complete and record the number of counts detected.

11. Prepare to collect the half-life data by setting the count frequency to 30.00 seconds.

12. The $^{137}\text{Ba}$ sample is prepared as follows (ask the instructor to do this):
   a) Obtain the generator kit and the bottle of eluting solution.
   b) Place a metal disk planchet in the larger, rectangular metal tray.
   c) Attach the plastic tube to the syringe and draw eluting solution into the syringe.
   d) Remove the tube from the syringe and remove the plastic stoppers (top and bottom) from the generator.
   e) Insert the syringe firmly into the hole on the top of the generator. While holding the generator over the planchet, **slowly** and **steadily** push on the syringe plunger to force solution through the generator until the planchet is full.
   f) Slide the planchet and tray under the G-M tube and **immediately** click ‘Record’.
   g) Click ‘Stop’ after an elapsed time of at least 12 minutes.

13. Export the data electronically:
   a) Go to the File menu and select ‘Export Data…’.
   b) Give the data file a descriptive name (e.g. halflife_data_20191015.txt).
   c) Open the tab-delimited text data file in Excel, save it as an Excel file, and distribute it to all the members of the lab group.

14. Once the experiment has been completed, log-off from the computer and turn off the PASCO 550 interface.

**Analysis:**

**Analysis Goals:**

1. Find the count rate for each data point (correcting for dead time and background).
2. Find the relationship between the count rates and elapsed time.
3. Determine the half-life of $^{137}\text{Ba}^m$.

When tabulating the data, convert the count rate (counts in 30 sec) to counts/sec. It may be necessary to correct for dead time at the high initial count rates. Use a value of 100 microseconds for the dead time for the G-M tube. Remember to also correct for background. Tabulate the corrected count rates (count/sec).
Plot the natural logarithm of the corrected count rates versus elapsed time.
Is the theoretical relation between count rate and elapsed time verified?
Determine the half-life of $^{137}\text{Ba}$ from your graph. Compare to the accepted value of 2.55 min (153 sec).

References:
- Bleuler & Goldsmith, Experimental Nucleonics, QC 784
- Fretter, Introduction to Experimental Physics, QC 41
- Halliday, Introductory Nuclear Physics, QC 173
- Melissinos, Experiments in Modern Physics, QC 33
- Taylor, An Introduction to Error Analysis, QA 275