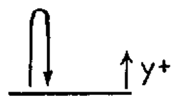
PART B

B1. A bullet is fired vertically upwards from the ground with a speed of 300 m/s. Find the time that the bullet is in the air.



$y_o = 0$

$y = 0$

$v_o = +300 \text{ m/s}$

$a = -9.80 \text{ m/s}^2$

$$y = y_o + v_o t + \frac{1}{2}at^2$$

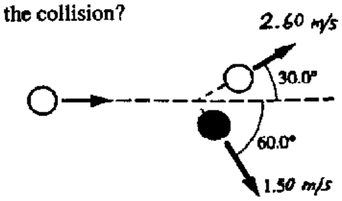
$$0 = v_o t + \frac{1}{2}at^2$$

$$0 = (v_o + \frac{1}{2}at)t$$

$$\therefore \quad v_o + \frac{1}{2}at = 0$$

$$t = -\frac{2v_o}{a} = \frac{-2(300 \text{ m/s})}{-9.80 \text{ m/s}^2} = 61.2 \text{ s}$$

B2. Billiard balls have a mass of 0.170 kg. The diagram shows a billiard ball (black) being struck by a cue ball (white). What is the speed of the cue ball before the collision?



$$\vec{p}_o = \vec{p}_f$$

$$p_{ox} = p_{fx}$$

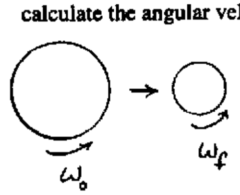
$$m v_{c_o} = m v_{c_f} \cos 30.0° + m v_{b_f} \cos 60.0°$$

$$v_{c_o} = v_{c_f} \cos 30.0° + v_{b_f} \cos 60.0°$$

$$v_{c_o} = (2.60 \text{ m/s}) \cos 30.0° + (1.50 \text{ m/s}) \cos 60.0°$$

$$v_{c_o} = 3.00 \text{ m/s}$$

B3. A spherical star of mass 2.00×10^{31} kg and radius 7.50×10^8 m is rotating with an angular velocity of 0.105 rad/s. Internal forces cause the star to collapse to a new radius of 1.25×10^8 m. Assuming the star to be a uniform sphere before and after the collapse, calculate the angular velocity of the collapsed star.



Ang. mom. is conserved.

$$L_o = L_f$$

$$I_o \omega_o = I_f \omega_f$$

$$\left(\frac{2}{5}MR_o^2\right)\omega_o = \left(\frac{2}{5}MR_f^2\right)\omega_f$$

$$\omega_f = \frac{R_o^2}{R_f^2} \cdot \omega_o = \frac{(7.50 \times 10^8 \text{ m})^2}{(1.25 \times 10^8 \text{ m})^2} \cdot 0.105 \text{ rad/s} = 3.78 \text{ rad/s}$$

B4. For testing its suitability for space flight, a computer is bolted to a spring-mounted platform as shown. The platform is made to oscillate in simple harmonic motion with an amplitude of 2.50 cm. Calculate the oscillation frequency f that gives a maximum acceleration of $15.0 \times g$.

$$a = -A\omega^2 \cos \omega t \; ; \quad \omega = 2\pi f$$

$$a_{max} = A\omega^2 = A(2\pi f)^2 = 4\pi^2 A f^2$$

$$f = \frac{1}{2\pi}\sqrt{\frac{a_{max}}{A}} = \frac{1}{2\pi}\sqrt{\frac{15.0(9.80 \text{ m/s}^2)}{0.0250 \text{ m}}}$$

$$f = 12.2 \text{ Hz}$$

B5. A string of mass 1.20 g and length 8.50 m is under a tension of 1.60 N. The string is oscillating in a standing wave pattern with nodes at each end and two nodes between its ends. Calculate the frequency of vibration of the string.

$L = \frac{3}{2}\lambda$

so $\lambda = \frac{2}{3}L$

and $v = \sqrt{\frac{F}{m/L}}$

$$f = \frac{v}{\lambda} = \frac{\sqrt{\frac{F}{m/L}}}{\frac{2}{3}L} = \frac{3}{2}\sqrt{\frac{F}{mL}}$$

$$f = \frac{3}{2}\sqrt{\frac{1.60\text{N}}{(1.20\times10^{-3}\text{kg})(8.50\text{m})}}$$

$$f = 18.8\ \text{Hz}$$

B6. A very small speaker, with a power output of 100 W, emits sound uniformly in all directions. What is the sound intensity at a distance of 10.0 m from the speaker?

$$I = \frac{P}{A} = \frac{P}{4\pi R^2} = \frac{100\text{W}}{4\pi(10.0\text{m})^2} = 7.96\times10^{-2}\ \text{W/m}^2$$

B7. A diamond ring is placed 30.0 cm in front of a lens. The image of the ring is inverted and 0.400 times the size of the ring. Find the focal length of the lens.

Inverted image means converging lens.

$d_o = 30.0\text{cm}$

$m = \frac{-d_i}{d_o} = -0.400$

$\therefore -d_i = -0.400(30.0\text{cm})$

$d_i = 12.0\text{cm}$

$$f = \left(\frac{1}{d_o} + \frac{1}{d_i}\right)^{-1}$$

$$f = \left(\frac{1}{30.0\text{cm}} + \frac{1}{12.0\text{cm}}\right)^{-1}$$

$$f = 8.57\text{cm}$$

B8. When light from a mercury source passes through a diffraction grating, the third-order maximum of the 578-nm yellow line occurs at the same location as the fourth-order maximum of a different wavelength. Calculate the wavelength of the fourth-order line.

$\sin\theta = \frac{m\lambda}{d}$ same location means $m_1\lambda_1 = m_2\lambda_2$

$$\therefore \lambda_2 = \frac{m_1\lambda_1}{m_2} = \frac{3(578\text{nm})}{4} = 434\text{nm}$$

B9. The work function for a particular metal is 4.20 eV. What is the minimum frequency of photons needed to eject electrons from the metal surface with a maximum kinetic energy of 1.00 eV?

$$hf = KE_{max} + W_o$$

$$f = \frac{KE_{max} + W_o}{h} = \frac{1.00\,eV + 4.20\,eV}{4.14 \times 10^{-15}\,eV \cdot s}$$

$$f = 1.26 \times 10^{+15}\ Hz$$

B10. Calculate the binding energy (in MeV) of $^{23}_{11}Na$ (atomic mass = 22.989767 u).

$$BE = \left[Z m_H + N m_n - m\left(^{23}_{11}Na\right) \right] \cdot c^2$$

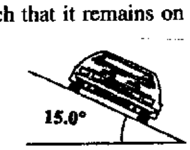
$$BE = \left[11(1.007825\,u) + 12(1.008665\,u) - 22.989767\,u \right] \cdot 931.5 \frac{MeV}{u}$$

$$BE = 187\ MeV$$

PART C

C1. A car of mass 1500 kg is travelling around a banked curve which is inclined at an angle of 15.0° above the horizontal.

(a) The car's path is circular with a radius of curvature of 100 m. Assuming that the road surface is frictionless, calculate the speed of the car such that it remains on the banked curve.

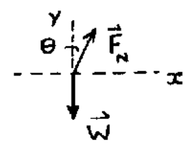


$\boxed{16.2 \text{ m/s}}$

In y-dir'n: $\sum F_y = 0 \Rightarrow F_N \cos\theta - W = 0$

In x-dir'n: $\sum F_x = m a_c$

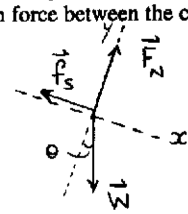
$F_N \sin\theta = m\frac{v^2}{r}$



$v^2 = \frac{F_N \sin\theta\, r}{m}$ and $F_N = \frac{W}{\cos\theta} = \frac{mg}{\cos\theta}$

so $v = \sqrt{\frac{\cancel{m}g}{\cos\theta}\frac{\sin\theta\, r}{\cancel{m}}} = \sqrt{rg\tan\theta} = \sqrt{100\text{m}(9.80\text{m/s}^2)\tan 15.0°}$

(b) Real road surfaces are not frictionless. Suppose the driver stops the car on the banked curve. Calculate the magnitude of the total static friction force between the car's tires and the road surface.



$\boxed{3.80 \times 10^3 \text{ N}}$

$\sum F_x = 0$

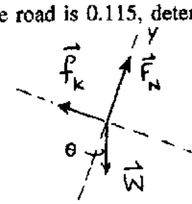
$W\sin\theta - f_s = 0$

$mg\sin\theta = f_s$

$(1500\text{kg})(9.80\text{m/s}^2)\sin 15.0° = f_s$

$f_s = 3805\text{N}$

(c) It starts to rain and the car slides sideways down the inclined road surface. If the coefficient of kinetic friction between the tires and the road is 0.115, determine the acceleration of the car as it slides.



$\boxed{1.45 \text{ m/s}^2}$

$\sum F_y = 0 \Rightarrow F_N - W\cos\theta = 0$

$F_N = mg\cos\theta$

$\sum F_x = ma$

$W\sin\theta - f_k = ma$ so $mg\sin\theta - \mu_k F_N = ma$

$ma = mg\sin\theta - \mu_k mg\cos\theta$

$a = g\sin\theta - \mu_k g\cos\theta = 9.80\text{m/s}^2(\sin 15.0° - 0.115\cos 15.0°)$

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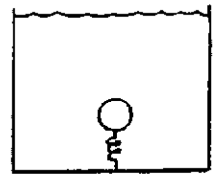
C2. A ball of mass 0.450 kg is anchored to the bottom of a swimming pool by an ideal spring of force constant 60.0 N/m attached to the pool's bottom. The spring stretches 4.20 cm from its unstretched length as the ball reaches equilibrium at a depth of 3.50 m. The density of water is 1000 kg/m^3.



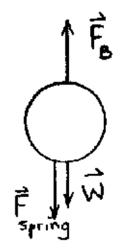
(a) Calculate the magnitude of the force exerted on the ball by the spring.

$\boxed{2.52\,N}$

$$|F| = kx$$

$$|F| = (60.0\,N/m)(0.0420\,m)$$

$$|F| = 2.52\,N$$



(b) Calculate the radius of the ball.

$\boxed{5.53 \times 10^{-2}\,m}$

$$\Sigma \vec{F} = 0$$

$$F_B - mg - F_{spring} = 0$$

$$\rho_{H_2O}\, g V - mg - F_{spring} = 0$$

$$V = \frac{mg + F_{spring}}{\rho_{H_2O}\, g} = \frac{4}{3}\pi r^3$$

$$r = \left(\frac{3}{4\pi}\left(\frac{mg + F_{spring}}{\rho_{H_2O}\, g}\right)\right)^{1/3} = 0.0553\,m$$

(c) Assuming the atmospheric pressure is 1.02×10^5 Pa, calculate the absolute pressure at the depth of the ball.

$\boxed{1.36 \times 10^5\,Pa}$

$$P_2 = P_1 + \rho g h = 1.02 \times 10^5\,Pa + (1000\,kg/m^3)(9.80\,m/s^2)\cdot(3.50\,m)$$

$$P_2 = 1.36 \times 10^5\,Pa$$

C3. The energy released when a radioisotope decays may be used to produce electricity to power such things as interplanetary probes. Consider the radioactive nucleus $^{210}_{84}Po$ (polonium) which decays by alpha emission to $^{206}_{82}Pb$ (lead) with a half-life of 138.4 days.

Atomic mass of $^{210}_{84}Po$ = 209.982848 u
$^{206}_{82}Pb$ = 205.974440 u
$^{4}_{2}He$ = 4.002603 u

(a) Calculate the energy released (in MeV) in the decay of each $^{210}_{84}Po$ atom.

$\boxed{5.41 \text{ MeV}}$ $Q = \Delta m \cdot c^2$

$$Q = [209.982848\,u - (205.974440\,u + 4.002603\,u)] \cdot 931.5 \frac{\text{MeV}}{u}$$

$$Q = 5.41 \text{ MeV}$$

(b) If a 1.00-g sample of $^{210}_{84}Po$ initially contains 2.87×10^{21} atoms, what is the initial decay rate from this sample (in Bq)?

$\boxed{1.66 \times 10^{14} \text{ Bq}}$

$$A = \lambda N \;;\quad \lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{138.4\,d \times \frac{24\,h}{d} \times \frac{3600\,s}{h}}$$

$$\lambda = 5.80 \times 10^{-8}\,s^{-1}$$

$$A = (5.80\times10^{-8}\,s^{-1})(2.87\times10^{21}) = 1.66\times10^{14} \text{ Bq}$$

(c) How many of this initial number of atoms remain after one year?

$\boxed{4.61 \times 10^{20}}$ $N = N_0 e^{-\lambda t}$

$$N = 2.87\times10^{21}\, e^{-5.80\times10^{-8}\,s^{-1}\left(1\,y \times 365\frac{d}{y} \times \frac{24\,h}{d} \times \frac{3600\,s}{h}\right)}$$

$$N = 4.61 \times 10^{20}$$

(d) What is the average power output (in W) from the decay of $^{210}_{84}Po$ in this sample during the first year?

$\boxed{66.1 \text{ W}}$ # of decays = $N - N_0$

$$P = \frac{\#\text{ of decays}}{\text{time}} \times \frac{\text{energy}}{\text{decay}} = \frac{2.87\times10^{21} - 4.61\times10^{20}}{1\,y \times 365\frac{d}{y} \times \frac{24\,h}{d} \times \frac{3600\,s}{h}} \times 5.41 \text{ MeV}$$

$$P = 4.13\times10^{14} \frac{\text{MeV}}{s} \times 1.60\times10^{-13} \frac{J}{\text{MeV}} = 66.1 \text{ W}$$