

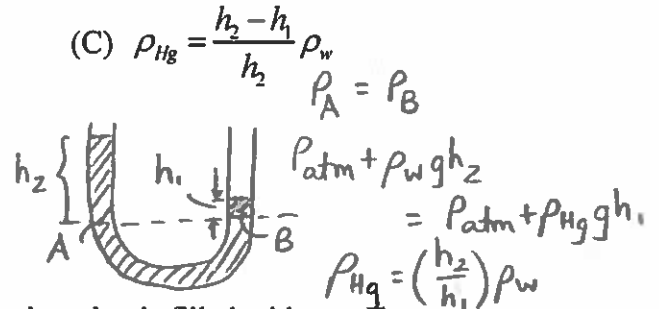
PART A

FOR EACH OF THE FOLLOWING QUESTIONS IN PART A, ENTER THE MOST APPROPRIATE RESPONSE ON THE OMR SHEET.

A1. A U-tube is partially filled with water. Mercury (which does not mix with water) is then added to the right side of the tube. The top of the mercury is a distance h_1 above the level of the interface between the mercury and water. On the left side of the tube the top of the water is a distance h_2 above the level of the mercury-water interface on the right side. What is the density of mercury, ρ_{Hg} , in terms of the density of water, ρ_w ?

B

- (A) $\rho_{Hg} = \frac{h_1}{h_2} \rho_w$ (B) $\rho_{Hg} = \frac{h_2}{h_1} \rho_w$
 (D) $\rho_{Hg} = \frac{h_1}{h_2 - h_1} \rho_w$ (E) $\rho_{Hg} = \frac{h_2 - h_1}{h_2 + h_1} \rho_w$



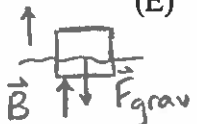
A2. Two solid objects of identical mass are placed in a container that is filled with an unknown liquid. One object floats and the other sinks to the bottom. Which one of the following is a true statement concerning the volumes of the objects?

B

- (A) Both objects have the same volume.
 (B) The floating object's volume is greater than the volume of the object that sinks.
 (C) The floating object's volume is less than the volume of the object that sinks.
 (D) Nothing can be said about the volumes without knowing the densities of the objects.
 (E) Nothing can be said about the volumes without knowing the density of the unknown liquid.

$\therefore \rho_s > \rho_f \Rightarrow \frac{m}{V_s} > \frac{m}{V_f} \Rightarrow V_f > V_s$

FLOATER:



$B - F_{grav} = 0$
 $B = mg$
 $\rho_l g V_{dis} = \rho_f g V_f$
 $V_{dis} < V_f \Rightarrow \rho_l > \rho_f$

SUBMERGED:

$B < F_{grav}$
 $\rho_l g V_s < \rho_s g V_s \Rightarrow \rho_s > \rho_l$

A3. An ideal fluid flows through a pipe made of two sections with diameters of 1.0 cm and 4.0 cm, respectively. How is the speed of the fluid flow through the 4.0-cm section, v_4 , related to the speed of the fluid flow through the 1.0-cm section, v_1 ?

A

- (A) $v_4 = \frac{1}{16} v_1$ (B) $v_4 = \frac{1}{4} v_1$ (C) $v_4 = \frac{1}{2} v_1$ (D) $v_4 = 4 v_1$ (E) $v_4 = 16 v_1$

Continuity Equation: $A_1 v_1 = A_4 v_4$
 $\pi \left(\frac{d_1}{2}\right)^2 v_1 = \pi \left(\frac{d_4}{2}\right)^2 v_4$

$v_4 = \left(\frac{d_1}{d_4}\right)^2 v_1 = \left(\frac{1.0 \text{ cm}}{4.0 \text{ cm}}\right)^2 v_1 = \frac{1}{16} v_1$

A4. Which one of the following quantities is at a maximum when an object in simple harmonic motion is at its maximum displacement?

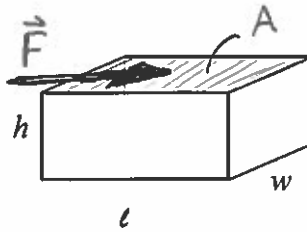
A

- (A) acceleration (B) speed (C) momentum
 (D) kinetic energy (E) frequency

At max. displacement, $v = 0$ $\therefore \vec{p} = m\vec{v}$ and $KE = \frac{1}{2} m v^2$ are also 0
 frequency is constant $\vec{a} = -\frac{k}{m} \vec{x} \Rightarrow |\vec{a}|$ is max. when $|\vec{x}|$ is max.

5. A rectangular block has dimensions h , l , and w , as shown in the diagram below. If a force of magnitude F is applied parallel to the top surface of the block, which one of the following expressions is correct for the shear stress exerted on the top surface of the block?

B



$$\text{Shear Stress} = \frac{F}{A_{\text{parallel}}} = \frac{F}{lw}$$

- (A) $\frac{F}{hl}$ (B) $\frac{F}{wl}$ (C) $\frac{F}{hw}$ (D) $\frac{F}{l^2}$ (E) $\frac{F}{w^2}$

- A6. Due to a build-up of sludge, the effective radius of an ^{horizontal} oil pipeline becomes half the original radius. To compensate for this reduced radius, the pipeline operator increases the pressure difference across the length of the pipeline by a factor of four. If Q_1 is the original volume flow rate through the pipeline, what is the new volume flow rate, Q_2 , in terms of Q_1 ? (η does not change)

E

- (A) $Q_2 = 4 Q_1$ (B) $Q_2 = 2 Q_1$ (C) $Q_2 = Q_1$
(D) $Q_2 = \frac{1}{2} Q_1$ (E) $Q_2 = \frac{1}{4} Q_1$

$$Q_1 = \frac{\pi R_1^4 (P_1 - P_2)_1}{8\eta L} ; \quad Q_2 = \frac{\pi R_2^4 (P_1 - P_2)_2}{8\eta L} = \frac{\pi (R_1/2)^4 (4(P_1 - P_2)_1)}{8\eta L} = \frac{4}{16} \frac{\pi R_1^4 (P_1 - P_2)_1}{8\eta L}$$

- A7. If one could transport a simple pendulum of constant length from the Earth's surface to the Moon's, where the acceleration due to gravity is one-sixth ($1/6$) of that on Earth, by what factor would the pendulum frequency be changed?

C

- (A) $f_M \approx 6f_E$ (B) $f_M \approx 2.5f_E$ (C) $f_M \approx 0.41f_E$ (D) $f_M \approx 0.17f_E$ (E) $f_M = 3.5f_E$

$$T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{L}} ; \quad f_M = \frac{1}{2\pi}\sqrt{\frac{g_M}{L}} = \frac{1}{2\pi}\sqrt{\frac{1/6 g_E}{L}} = \sqrt{\frac{1}{6}} \left(\frac{1}{2\pi}\sqrt{\frac{g_E}{L}} \right)$$

- A8. Which one of the following pairs of quantities do you need to know in order to calculate the wavelength of a travelling wave?

D

- (A) frequency and period (B) speed and amplitude (C) amplitude and frequency
(D) frequency and speed (E) period and amplitude

$$v = f\lambda \Rightarrow \lambda = \frac{v}{f}$$

- A9. The speed of a wave in a stretched string is initially 50 m/s. What will be the new wave speed if the tension in the string is increased by 18%?

B

- (A) 50 m/s (B) 54 m/s (C) 21 m/s (D) 59 m/s (E) 45 m/s

$$v_1 = \sqrt{\frac{F_1}{\mu}} ; \quad v_2 = \sqrt{\frac{F_2}{\mu}} = \sqrt{\frac{1.18F_1}{\mu}} = 1.086\sqrt{\frac{F_1}{\mu}} = 1.086v_1 = 1.086(50\text{m/s}) = 54\text{m/s}$$

A10. How is the direction of propagation of an electromagnetic wave oriented relative to the directions of the associated electric and magnetic fields?

- C (A) parallel to the magnetic field, perpendicular to the electric field
 (B) perpendicular to the magnetic field, parallel to the electric field
 (C) perpendicular to the magnetic field, perpendicular to the electric field
 (D) parallel to the magnetic field, parallel to the electric field
 (E) parallel to the magnetic field, anti-parallel to the electric field

$$\vec{v} \perp \vec{B} \perp \vec{E}$$

$$\vec{v} \perp \vec{B}, \vec{E}$$

A11. It is observed that the air in a pipe resonates at frequencies of 120 Hz (the fundamental) and 600 Hz, and possibly other frequencies between these two values. If the pipe is open at both ends, how many additional resonant frequencies are there between 120 Hz and 600 Hz; and if the pipe is open at one end and closed at the other, how many additional resonant frequencies are there between 120 Hz and 600 Hz?

- A (A) open: 3 ; closed: 1 (B) open: 1 ; closed: 3 (C) open: 2 ; closed: 0
 (D) open: 0 ; closed: 2 (E) open: 5 ; closed: 1

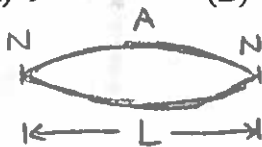
f_{open} : all harmonics ; f_{closed} : odd harmonics \Rightarrow

$$f_{open}: 120\text{ Hz}, 240\text{ Hz}, 360\text{ Hz}, 480\text{ Hz}, 600\text{ Hz}$$

$$f_{closed}: 120\text{ Hz}, 360\text{ Hz}, 600\text{ Hz}$$

A12. If the tension in a guitar string is increased by a factor of 3, by what factor does the fundamental frequency at which the string vibrates change?

- C (A) 9 (B) 3 (C) $\sqrt{3}$ (D) $\frac{1}{\sqrt{3}}$ (E) $\frac{1}{3}$



$$L = \frac{1}{2} \lambda_1 \quad \lambda_1 = 2L$$

$$f_1 = \frac{v}{\lambda_1} = \frac{1}{2L} \sqrt{\frac{F}{\mu}} ; f_1' = \frac{1}{2L} \sqrt{\frac{3F}{\mu}}$$

$$f_1' = \sqrt{3} \left(\frac{1}{2L} \sqrt{\frac{F}{\mu}} \right) = \sqrt{3} f_1$$

PART B

WORK OUT THE ANSWERS TO THE FOLLOWING PART B QUESTIONS.

WHEN YOU HAVE AN ANSWER THAT IS ONE OF THE OPTIONS AND ARE CONFIDENT THAT YOUR METHOD IS CORRECT, SCRATCH THAT OPTION ON THE SCRATCH CARD. IF YOU REVEAL A STAR ON THE SCRATCH CARD THEN YOUR ANSWER IS CORRECT (FULL MARKS, 2/2).

IF YOU DO NOT REVEAL A STAR WITH YOUR FIRST SCRATCH, TRY TO FIND THE ERROR IN YOUR SOLUTION. IF YOU REVEAL A STAR WITH YOUR SECOND SCRATCH, YOU RECEIVE HALF-MARKS (1/2).

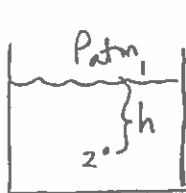
IF YOU STILL DO NOT HAVE THE CORRECT ANSWER, BUT REWORK YOUR SOLUTION AND REVEAL A STAR WITH YOUR THIRD SCRATCH, THEN YOU RECEIVE 0.2/2.

REVEALING THE STAR WITH YOUR FOURTH OR FIFTH SCRATCHES DOES NOT EARN YOU ANY MARKS, BUT IT DOES GIVE YOU THE CORRECT ANSWER.

YOU MAY ANSWER ALL FOUR PART B QUESTION GROUPINGS (1-4, 5-8, 9-12, AND 13-16) AND YOU WILL RECEIVE THE MARKS FOR YOUR BEST 3 GROUPINGS.

USE THE PROVIDED EXAM BOOKLET FOR YOUR ROUGH WORK.

- B1. A large storage tank is open to the atmosphere at the top and filled with water. What is the gauge pressure at a depth of 8.00 m below the surface in the tank of water?



$$P_2 = P_1 + \rho g(y_1 - y_2)$$

$$P_{2\text{ gauge}} = P_{2\text{ absolute}} - P_{atm}$$

$$P_2 = P_{atm} + \rho gh$$

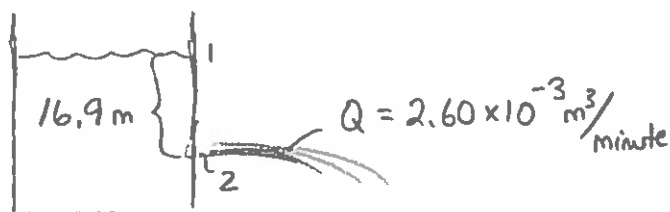
$$P_{2\text{ gauge}} = (P_{atm} + \rho gy) - P_{atm} = \rho gy$$

$$P_{2\text{ gauge}} = (1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(8.00 \text{ m}) = \boxed{7.84 \times 10^4 \text{ Pa}}$$

The tank develops a small hole in its side at a point 16.9 m below the surface of the water. The rate of flow from the leak is $2.60 \times 10^{-3} \text{ m}^3/\text{minute}$.

- B2. What is the pressure just outside the small hole in the side of the storage tank?

atmospheric pressure



- B3. Calculate the speed at which water leaves the hole. Assume that the surface area of water that is open to the atmosphere at the top of the tank is much larger than the area of the small hole.

Apply Bernoulli's between the top of the tank and the hole

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gy_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gy_2 ; P_1 = P_2 = P_{atm}$$

Continuity Equation: $A_1 v_1 = A_2 v_2$; given $A_1 \gg A_2 \Rightarrow v_1 \ll v_2$
 \therefore set $v_1 = 0$.

$$\therefore \rho gy_1 = \frac{1}{2}\rho v_2^2$$

$$v_2 = \sqrt{2gy_1} = \sqrt{2(9.80 \text{ m/s}^2)(16.9 \text{ m})} = \boxed{18.2 \text{ m/s}}$$

- B4. Calculate the radius of the hole.

$$Q = A_2 v_2 = \pi r_2^2 v_2$$

$$r_2 = \sqrt{\frac{Q}{\pi v_2}} = \left(\frac{(2.60 \times 10^{-3} \text{ m}^3/\text{min})(1 \text{ min}/60 \text{ s})}{\pi (18.2 \text{ m/s})} \right)^{1/2}$$

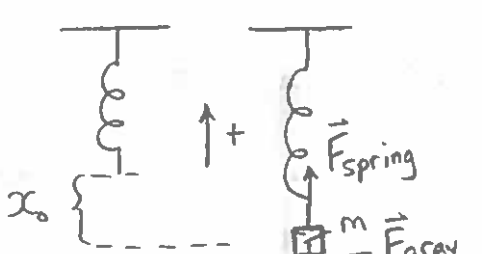
$$r_2 = 8.71 \times 10^{-4} \text{ m} = \boxed{0.871 \text{ mm}}$$

B5. Which one of the following statements correctly describes the type of force necessary for simple harmonic motion (SHM) to occur?

SHM occurs for any restoring force whose magnitude is proportional to the magnitude of the displacement from a point of stable equilibrium.

A vertical, massless spring is attached to the ceiling and is initially relaxed (neither stretched nor compressed). A block of mass $m = 1.10 \text{ kg}$ is now attached to the spring and gently lowered (stretching the spring) until the mass is at rest at its equilibrium position. When the mass is hanging at rest at its equilibrium position the spring is 1.70 cm longer than its relaxed length.

B6. Calculate the spring constant.



$$\sum \vec{F} = 0$$

$$+kx_0 = mg$$

$$k = \frac{mg}{x_0} = \frac{(1.10 \text{ kg})(9.80 \text{ m/s}^2)}{(1.70 \text{ cm})(1 \text{ m}/100 \text{ cm})} = 634.1 \text{ N/m}$$

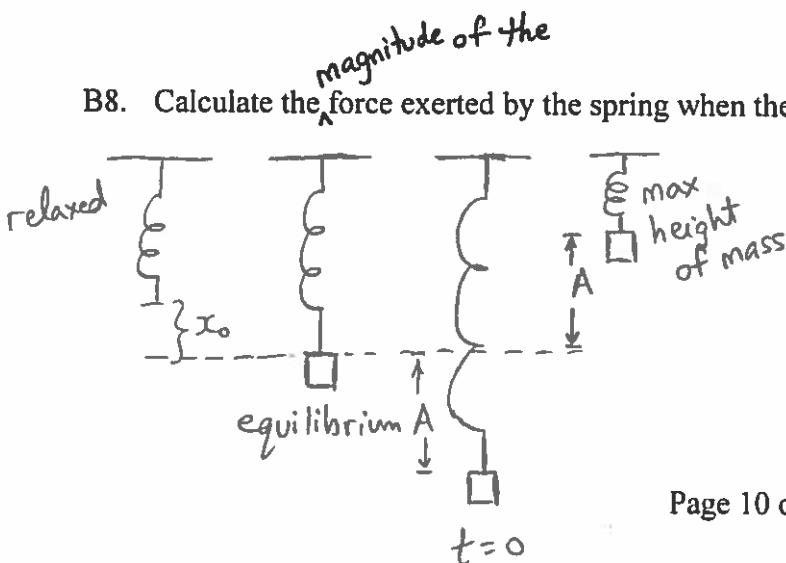
$$= \boxed{634 \text{ N/m}}$$

As you know from lab M19, a vertical mass-spring system will oscillate in SHM. Suppose that the 1.10-kg mass, initially at rest at its equilibrium position, is pulled down a distance of 6.00 cm and released.

B7. Calculate the period of the oscillations of the mass-spring system.

$$\frac{2\pi}{T} = \sqrt{\frac{k}{m}} \Rightarrow T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{(1.10 \text{ kg})}{634.1 \text{ N/m}}} = \boxed{0.262 \text{ s}}$$

B8. Calculate the magnitude of the force exerted by the spring when the mass is at its maximum height.



At max. height, the mass will be 6.00 cm above its equilibrium position. The spring will therefore be compressed a distance $A - x_0 = x$

$$x = 6.00 \text{ cm} - 1.70 \text{ cm} = 4.30 \text{ cm}$$

$$|\vec{F}_{\text{spring}}| = k|x|$$

$$= (634.1 \frac{\text{N}}{\text{m}})(0.0430 \text{ m})$$

$$|\vec{F}_{\text{spring}}| = \boxed{27.3 \text{ N}}$$

$$v = 331 \text{ m/s} \sqrt{\frac{293 \text{ K}}{273 \text{ K}}} = 343 \text{ m/s}$$

On a completely calm day, when the air temperature is 20.0°C , two people are in a stationary hot air balloon at an altitude of 595 m. A small plane is approaching the balloon, at a constant speed of 224 km/h. The frequency of the sound emitted by the engine of the plane is 155 Hz.

B9. Which one of the following statements correctly describes how the intensity and frequency of the plane's sound is perceived by the balloon's occupants as the plane approaches?

The frequency is constant at a value greater than 155 Hz, and the intensity increases, as the plane approaches.

B10. When the plane is at a distance of 5.00 km from the balloon, the intensity level of the plane's sound, as heard by the occupants of the balloon, is 60.0 dB. What is the intensity level heard by the balloon's occupants when the plane is at a distance of 0.500 km?

$$P = IA \Rightarrow I_1 A_1 = I_2 A_2 \Rightarrow I_1 4\pi r_1^2 = I_2 4\pi r_2^2 \Rightarrow I_2 = I_1 \left(\frac{r_1}{r_2}\right)^2$$

$$I_2 = I_1 \left(\frac{5.00 \text{ km}}{0.500 \text{ km}}\right)^2 = 100 I_1 \quad I_2 \text{ is a factor of } 100 = 10^2 \text{ times } I_1$$

$$\therefore \beta_2 \text{ is } 2 \times 10 \text{ dB greater than } \beta_1$$

$$\beta_2 = 60.0 \text{ dB} + 20.0 \text{ dB}$$

B11. Calculate the intensity of the plane's sound, in W/m^2 , at the location of the balloon, when the plane is at a distance of 0.500 km.

$$\beta_2 = 10 \text{ dB} \log_{10} \left(\frac{I_2}{I_0}\right) \Rightarrow I_2 = I_0 \times 10^{\beta_2/10 \text{ dB}}$$

$$\beta_2 = 80.0 \text{ dB}$$

$$I_2 = (1.00 \times 10^{-12} \text{ W/m}^2) 10^{80.0 \text{ dB}/10 \text{ dB}} = 1.00 \times 10^{-4} \text{ W/m}^2$$

B12. Calculate the change in the frequency of the plane's sound, as detected by an occupant of the balloon, between when the plane is approaching the balloon and when the plane has passed by the balloon and is moving further away.

moving source. $f_o = f_s \left(\frac{v}{v - v_s}\right)$

$$|v_s| = 224 \frac{\text{km}}{\text{h}} \times \frac{1 \text{ h}}{3600 \text{ s}} \times \frac{1000 \text{ m}}{\text{km}}$$

$$= 62.2 \text{ m/s}$$

$$\Delta f = f_{\text{approach}} - f_{\text{away}} = f_s v \left(\frac{1}{v - v_{s_{\text{app}}}} - \frac{1}{v - v_{s_{\text{away}}}} \right)$$

$$\Delta f = (155 \text{ Hz})(343 \text{ m/s}) \left(\frac{1}{343 \text{ m/s} - (+62.2 \text{ m/s})} - \frac{1}{343 \text{ m/s} - (-62.2 \text{ m/s})} \right)$$

$$\Delta f = 58.1 \text{ Hz}$$

B13. Consider two pipes of equal length. One pipe is open at both ends and the other pipe is open at one end and closed at the other. How do the fundamental frequencies of the two pipes compare?

$$\frac{1}{2}\lambda_o \left\{ \begin{array}{l} A \\ N \\ A \end{array} \right. \left. \begin{array}{l} \uparrow \\ L \\ \downarrow \end{array} \right. \left. \begin{array}{l} N \\ A \end{array} \right\} \frac{1}{4}\lambda_c \quad \frac{1}{2}\lambda_o = \frac{1}{4}\lambda_c$$

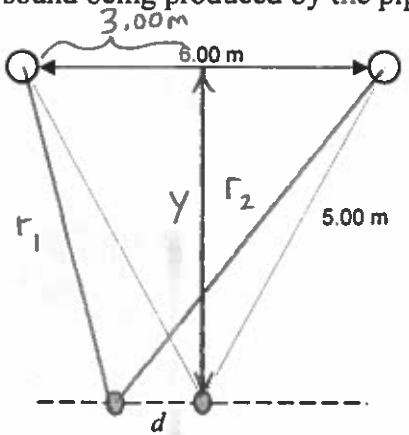
$$\frac{1}{2}\left(\frac{v}{f_o}\right) = \frac{1}{4}\left(\frac{v}{f_c}\right)$$

Now consider two identical pipes. Both pipes are open at one end and closed at the other.

$$2f_o = 4f_c$$

B14. The two pipes are separated by a distance of 6.00 m and both are producing sound at the fundamental frequency. A listener is initially standing in a position such that she is 5.00 m from each pipe. The listener then moves a distance of $d = 0.629$ m along the line shown in the diagram, and she is now at the first position of destructive interference. Calculate the wavelength of the sound being produced by the pipes.

$$f_c = \frac{1}{2}f_o$$



$$r_2 - r_1 = \frac{1}{2}\lambda \Rightarrow \lambda = 2(r_2 - r_1)$$

note that $y^2 = (5.00\text{m})^2 - (3.00\text{m})^2 \Rightarrow y = 4.00\text{m}$

$$r_1 = \sqrt{(3.00\text{m} - d)^2 + (4.00\text{m})^2} = 4.65\text{m}$$

$$r_2 = \sqrt{(3.00\text{m} + d)^2 + (4.00\text{m})^2} = 5.40\text{m}$$

$$\lambda = 2(5.40\text{m} - 4.65\text{m}) = 1.50\text{m}$$

B15. Calculate the length of each pipe.

From the work for B13, $L = \frac{1}{4}\lambda_c = \frac{1}{4}(1.50\text{m}) = 0.375\text{m}$

B16. If the air temperature in one of the pipes increases to 26.0°C, and the air temperature in the other pipe remains at 20.0°C, calculate the beat frequency that is heard when both pipes are producing sound at their fundamental frequencies.

$$v_{20.0} = (331\text{m/s})\sqrt{\frac{293\text{K}}{273\text{K}}} = 343\text{m/s} \quad ; \quad v_{26.0} = (331\text{m/s})\sqrt{\frac{299\text{K}}{273\text{K}}} = 346.4\text{m/s}$$

$$f_{\text{beat}} = |f_2 - f_1| = \left| \frac{v_{26}}{\lambda} - \frac{v_{20}}{\lambda} \right| = \frac{1}{1.50\text{m}} (346.4\text{m/s} - 343\text{m/s})$$

$$f_{\text{beat}} = 2.3\text{Hz}$$

END OF EXAMINATION