

UNIVERSITY OF SASKATCHEWAN

Department of Physics and Engineering Physics

Physics 115.3 MIDTERM TEST

October 20, 2016

Time: 90 minutes

NAME: SOLUTIONS MASTER
(Last) **Please Print** (Given)

STUDENT NO.: _____


LECTURE SECTION (please check):

- 01 A. Zulkoskey
- 02 Dr. D. Janzen
- 03 B. Zulkoskey
- 04 Dr. S. Litt
- 97 Dr. A. Farahani
- C15 Dr. A. Farahani

INSTRUCTIONS:

1. This is a closed book exam.
2. The test package includes a test paper (this document), a formula sheet, and an OMR sheet. The test paper consists of 8 pages, including this cover page. **It is the responsibility of the student to check that the test paper is complete.**
3. Only a basic scientific calculator (e.g. Texas Instruments TI-30X series, Hewlett-Packard HP 10s or 30S) may be used. Graphing or programmable calculators, or calculators with communication capability, are **not** allowed.
4. Enter your name and student number on the cover of the test paper and check the appropriate box for your lecture section. Also enter your student number in the top right-hand corner of each page of the test paper.
5. Enter your name and NSID on the OMR sheet.
6. The test paper, the formula sheet and the OMR sheet must all be submitted.
7. The marked test paper will be returned. The formula sheet and the OMR sheet will **NOT** be returned.

***ONLY THE THREE PART B QUESTIONS THAT YOU INDICATE WILL BE MARKED
PLEASE INDICATE WHICH THREE PART B QUESTIONS ARE TO BE MARKED***



QUESTION NUMBER	TO BE MARKED	MAXIMUM MARKS	MARKS OBTAINED
A1-15	<input checked="" type="checkbox"/>	15	
B1	<input type="checkbox"/>	10	
B2	<input type="checkbox"/>	10	
B3	<input type="checkbox"/>	10	
B4	<input type="checkbox"/>	10	
TOTAL		45	

PART A

FOR EACH OF THE FOLLOWING QUESTIONS IN PART A, ENTER THE MOST APPROPRIATE RESPONSE ON THE OMR SHEET.

- D A1. The length and width of a standard sheet of paper is measured, and the area is found by calculation to be 93.50 cm². The length has five significant figures in its measurement. The number of significant figures in the width measurement must be:
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

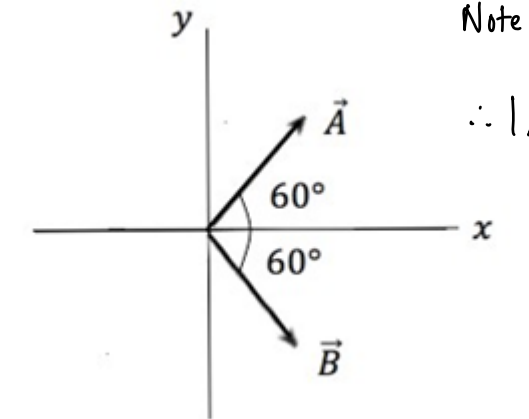
- D A2. Given that the dimensions of f , m , and k are as follows:
 $f : \frac{1}{[T]}$ $m : [M]$ $k : \frac{[M]}{[T]^2}$

determine which one of the following equations is dimensionally correct.

(A) $f = \frac{1}{2\pi} \frac{m}{k}$ (B) $f = 2\pi \frac{k}{m}$ (C) $f = \frac{1}{2\pi} \sqrt{\frac{m}{k}}$ (D) $f = 2\pi \sqrt{\frac{k}{m}}$ (E) $f = 2\pi \sqrt{mk}$

$\frac{[M]}{[M]/[T]^2} = [T]^2$ $\frac{[M]}{[M]} = \frac{1}{[T]^2}$ $[T]$ $\frac{1}{[T]}$ $\sqrt{\frac{[M][M]}{[T]^2}} = \frac{[M]}{[T]}$

- A3. Vectors \vec{A} and \vec{B} have the same magnitude of 2.0 cm and make angles of 60.0° with the positive x axis as shown in the diagram. What is the magnitude of the vector sum, $\vec{A} + \vec{B}$?



Note that $B_y = -A_y$
 $\therefore |\vec{A} + \vec{B}| = A_x + B_x$
 $= 2.0 \text{ cm } \cos(60.0^\circ) + 2.0 \text{ cm } \cos(60.0^\circ)$
 $= 2.0 \text{ cm}$

- A (A) 2.0 cm (B) 2.3 cm (C) 3.0 cm (D) 3.5 cm (E) 4.0 cm

- C A4. A racing car starts from rest and reaches a final speed v in a time t . If the acceleration of the car is constant during this time, which one of the following statements must be true?
(A) The car travels a distance vt .
(B) The acceleration of the car is vt .
(C) The average speed of the car over the time interval t is $v/2$.
(D) The velocity of the car remains constant.
(E) The velocity of the car increases exponentially.

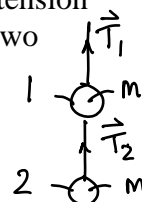
$\Delta x = \frac{1}{2}(v_0 + v)t = \frac{1}{2}vt$
 $a = \frac{v - v_0}{t} = \frac{v}{t}$
 $\bar{v} = \frac{v_0 + v}{2} = \frac{v}{2}$

- C A5. A projectile is launched from Earth's surface at a certain initial velocity at an angle above the horizontal, reaching maximum height after time t_{\max} . Another projectile is launched with the same initial velocity and angle from the surface of the Moon, where the acceleration of gravity is one-sixth that of Earth. Neglecting air resistance (on Earth) and variations in the acceleration of gravity with height, how long does it take the projectile on the Moon to reach its maximum height?
 $v_{0y} = v_0 \sin \theta_0$ At max. height, $v_y = 0$; $v_y = v_{0y} + a_y t \Rightarrow t_{\max} = \frac{0 - v_0 \sin \theta_0}{a_y}$
(A) t_{\max} (B) $t_{\max} / 6$ (C) $6 t_{\max}$ (D) $\sqrt{6} t_{\max}$ (E) $36 t_{\max}$
 $t_{\max_E} = \frac{-v_0 \sin \theta_0}{-g}$; $t_{\max_M} = \frac{-v_0 \sin \theta_0}{-1/6 \cdot g} = 6 \frac{v_0 \sin \theta_0}{g} = 6 t_{\max}$

- E A6. A ball is thrown vertically upward. Which one of the following statements best describes the acceleration of the ball at the instant it reaches maximum height? You may assume that the ball is in free-fall while it is in the air. For free-fall, acceleration is g downward.
(A) zero (B) magnitude $< g$, directed upward
(C) magnitude $> g$, directed upward (D) magnitude $< g$, directed downward
(E) magnitude $= g$, directed downward

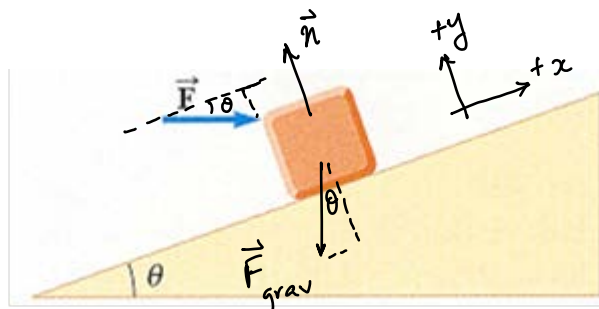
- A7. As a result of the coordinate system chosen to describe its motion, a car in straight line motion has a negative velocity and a positive acceleration. Which one of the following statements could possibly be correct for the motion of the car?
- negative velocity means moving in the negative direction.
acceleration with opposite sign as velocity means decreasing speed.*
- (A) The car is moving in the negative direction with increasing speed.
(B) The car is moving in the positive direction with increasing speed.
(C) The car is moving in the positive direction with decreasing speed.
(D) The car is moving in the negative direction with decreasing speed.
(E) The car is moving in the negative direction with constant speed.

- A8. Two monkeys of equal mass are holding onto a single vine of negligible mass that hangs vertically from a tree branch, with one monkey a few metres higher than the other. If the tension in the vine above the upper monkey is 400 N, what is the tension in the vine between the two monkeys?
- (A) 100 N **(B)** 200 N (C) 300 N (D) 400 N (E) 800 N



- A9. Consider the figure below. What is the magnitude of the resultant force exerted by the two cables supporting the traffic light?
- A*
- $\vec{T}_{1x} + \vec{T}_{2x} = 0$
 $\therefore |\vec{T}_1 + \vec{T}_2| = T_{1y} + T_{2y}$
 $= T_1 \sin 40.0^\circ + T_2 \sin 40.0^\circ$
 $= 70.7 \text{ N}$
-
- $T_1 = 55.0 \text{ N}$ $55.0 \text{ N} = T_2$
- B*
- $\vec{T}_1 + \vec{T}_2 - mg = 0$
 $T_1 - T_2 - mg = 0$
 $T_1 = T_2 + mg$
 $\therefore T_1 = 2mg = 2T_2$
 $T_2 = \frac{1}{2}T_1 = 200 \text{ N}$
- (A) 70.7 N (B) 84.3 N (C) 92.3 N (D) 115 N (E) 131 N

- A10. A block of mass m is held in equilibrium on a frictionless incline of angle θ by a horizontal force \vec{F} applied in the direction shown in the figure. What is the magnitude of \vec{F} required to hold the book in equilibrium?



$\Sigma \vec{F} = 0 \Rightarrow \Sigma F_x = 0 \text{ and } \Sigma F_y = 0$

- (A) mg (B) $mg \sin \theta$ (C) $mg \cos \theta$ **(D)** $mg \tan \theta$ (E) $mg \cot \theta$
- $\Sigma F_x = F \cos \theta - F_{\text{grav}} \sin \theta = 0 \Rightarrow F \cos \theta - mg \sin \theta = 0 \Rightarrow F = \frac{mg \sin \theta}{\cos \theta} = mg \tan \theta$

- A11. Imagine a spherical planet with eight times the mass of the Earth and exactly twice the radius. What would be the acceleration due to gravity near the surface of this planet?

- E*
- (A) 2.45 m/s^2 (B) 4.90 m/s^2 (C) 9.80 m/s^2 (D) 12.6 m/s^2 **(E)** 19.6 m/s^2
- On Earth, $mg = \frac{GM_E m}{R_E^2} \Rightarrow g = \frac{GM_E}{R_E^2}$ On planet, $mg' = \frac{G(8M_E)m}{(2R_E)^2} = \frac{8}{4} \frac{GM_E}{R_E^2} = 2 \frac{GM_E}{R_E^2} = 2g$

- A12. If the net work done on a particle is zero, which one of the following statements **must** be true?

- C*
- (A) The velocity of the particle is decreased.
(B) The velocity of the particle must be zero.
(C) The speed of the particle is unchanged.
(D) The velocity of the particle is unchanged.
(E) More information is required to answer the question.

$W_{\text{net}} = \Delta KE$

$W_{\text{net}} = 0 \Rightarrow \Delta KE = 0$

$\therefore KE$ is constant

$KE = \frac{1}{2}mv^2$, so speed must be constant

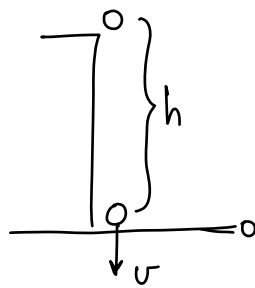
- A13. Two particles of different mass start from rest. The same net force acts on each of them as they move over equal distances. How do their final kinetic energies compare? $W_{net} = F_{net} \cos \theta d$
and $W_{net} = \Delta KE$
- D (A) The particle of larger mass has more kinetic energy.
(B) The particle of smaller mass has more kinetic energy.
(C) The particle of larger mass has less kinetic energy initially, but as its momentum increases its kinetic energy becomes greater than the less-massive particle's kinetic energy.
(D) The particles have equal kinetic energies. *same force and equal distance \Rightarrow Wnet same.*
(E) Not enough information is given to answer this question.

- A14. If two particles have equal kinetic energies, are their momenta equal? $\therefore \Delta KE$ same.
- C (A) Yes, always.
(B) Yes, as long as their masses are equal. $KE = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mKE}$
(C) Yes, if both their masses and directions of motion are the same. *equal KE \Rightarrow equal p if equal m.*
(D) No, unless they are moving perpendicular to each other.
(E) No, never.

but note that momentum is a vector.

- A15. A block of mass m is dropped from rest from the fourth floor of an office building, subsequently hitting the sidewalk at speed v . From what floor should the mass be dropped to double that impact speed? (The ground floor is "floor zero". The first floor is one floor above ground level.)

- E (A) The sixth floor
(B) The eighth floor
(C) The tenth floor
(D) The twelfth floor
(E) The sixteenth floor



$$E_f = E_i$$

$$KE_f + PE_f = KE_i + PE_i$$

$$\frac{1}{2}mv^2 + 0 = 0 + mgh$$

$$v = \sqrt{2gh} ; v_n = 2v_4$$

$$\sqrt{2gh_n} = 2\sqrt{2gh_4}$$

$$\sqrt{h_n} = 2\sqrt{h_4} \Rightarrow h_n = 4h_4 = 16^{th} \text{ floor}$$

PART B

ANSWER THREE OF THE PART B QUESTIONS ON THE FOLLOWING PAGES AND INDICATE ON THE COVER PAGE WHICH THREE PART B QUESTIONS ARE TO BE MARKED.

FOR EACH OF YOUR CHOSEN PART B QUESTIONS ON THE FOLLOWING PAGES, GIVE THE COMPLETE SOLUTION AND ENTER THE FINAL ANSWER IN THE BOX PROVIDED.

THE ANSWERS MUST CONTAIN THREE SIGNIFICANT FIGURES AND THE UNITS MUST BE GIVEN.

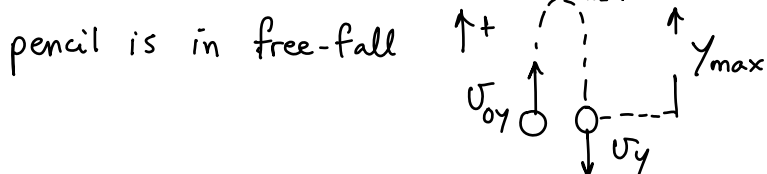
SHOW AND EXPLAIN YOUR WORK – NO CREDIT WILL BE GIVEN FOR ANSWERS ONLY.

EQUATIONS NOT PROVIDED ON THE FORMULAE SHEET MUST BE DERIVED.

USE THE BACK OF THE PREVIOUS PAGE FOR YOUR ROUGH WORK.

B1. During a midterm exam, you toss your pencil straight up in the air and catch it 2.00 s later at the same height as the point of release.

(a) What is the acceleration (magnitude and direction) of the pencil at maximum height? (1 mark)



$$\vec{a} = g \text{ downward}$$

(b) What is the speed of the pencil at maximum height? (1 mark)

Since the pencil is thrown straight up,
 $\vec{v} = 0$ at maximum height.

$$0$$

(c) Calculate the initial speed of the pencil. (2 marks)

$$t_{\text{tot}} = 2.00\text{s}; \Delta y = 0 \text{ (returns to initial height)}$$

$$9.80 \text{ m/s}$$

$$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$$

$$0 = v_{0y}t + \frac{1}{2}a_y t^2 \Rightarrow 0 = v_{0y} + \frac{1}{2}a_y t \Rightarrow v_{0y} = -\frac{1}{2}a_y t$$

$$v_{0y} = -\frac{1}{2}(-g)t$$

$$v_{0y} = \frac{1}{2}gt$$

$$v_{0y} = \frac{1}{2}(9.80 \text{ m/s}^2)(2.00\text{s}) = 9.80 \text{ m/s}$$

(d) Calculate the maximum height that the pencil reaches. If you did not obtain an answer for (c), use a value of 9.50 m/s. (3 marks)

$$v_y^2 = v_{0y}^2 + 2a_y \Delta y$$

$$4.90 \text{ m}$$

$$v_y = 0 \text{ at max. height}$$

$$\therefore 0 = v_{0y}^2 + 2a_y \Delta y_{\text{max}} \Rightarrow \Delta y_{\text{max}} = -\frac{v_{0y}^2}{2a_y} = -\frac{v_{0y}^2}{2(-g)}$$

$$\Delta y_{\text{max}} = \frac{-(9.80 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 4.90 \text{ m}$$

(e) If the pencil was only tossed half as high, how long would it be in the air? If you did not obtain an answer for (d), use a value of 5.00 m. (3 marks)

$$\Delta y_{\text{max}} = \frac{1}{2}(4.90 \text{ m}) = 2.45 \text{ m}$$

$$1.41 \text{ s}$$

$$v_y^2 = v_{0y}^2 + 2a_y \Delta y$$

$$0 = v_{0y}^2 + 2a_y \Delta y_{\text{max}} \Rightarrow v_{0y} = \sqrt{-2a_y \Delta y_{\text{max}}} = \sqrt{-2(-g)(\Delta y_{\text{max}})}$$

$$v_{0y} = \sqrt{2g \Delta y_{\text{max}}} = \sqrt{2(9.80 \text{ m/s}^2)(2.45 \text{ m})} = 6.93 \text{ m/s}$$

From symmetry of motion,
for whole flight $v_y = -v_{0y}$

$$\therefore v_y = v_{0y} + a_y t \Rightarrow -v_{0y} = v_{0y} - gt$$

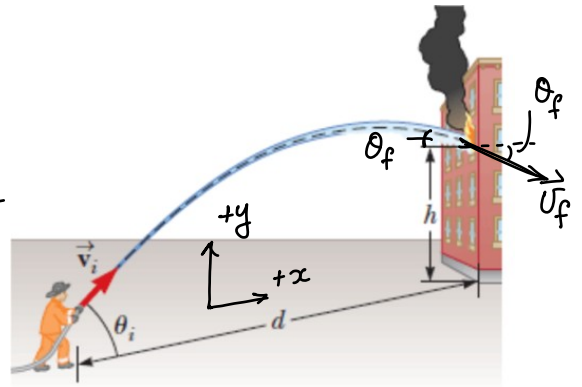
$$t = \frac{-2v_{0y}}{-g} = \frac{2(6.93 \text{ m/s})}{9.80 \text{ m/s}^2} = 1.41 \text{ s}$$

$$\text{or } \Delta y_{\text{tot}} = v_{0y}t_{\text{tot}} + \frac{1}{2}a_y t_{\text{tot}}^2$$

$$0 = v_{0y}t_{\text{tot}} + \frac{1}{2}(-g)t_{\text{tot}}^2$$

$$t_{\text{tot}} = \frac{-v_{0y}(2)}{-g} = 1.41 \text{ s}$$

B2. A fireman $d = 25.0$ m away from a burning building directs a stream of water from a ground-level fire hose at an angle of $\theta_i = 40.0^\circ$ above the horizontal as shown in the figure. It takes 1.63 s until the stream of water reaches the building.



Coordinate System: $+x$: horizontal to right
 $+y$: vertically upward

(a) Calculate the initial speed of the stream of water. (2 marks)

$$v_x = v_{ix} = v_i \cos \theta_i \quad (a_x = 0)$$

$$\Delta x = v_x t = (v_i \cos \theta_i) t$$

$$v_i = \frac{\Delta x}{(\cos \theta_i) t} = \frac{+25.0 \text{ m}}{(\cos 40.0^\circ)(1.63 \text{ s})} = 20.0 \text{ m/s}$$

20.0 m/s

(b) Calculate the height at which the stream of water strikes the building. If you did not obtain an answer for (a), use a value of 19.5 m/s. (3 marks)

$$\Delta y = v_{iy} t + \frac{1}{2} a_y t^2$$

7.96 m

$$\Delta y = (v_i \sin \theta_i) t + \frac{1}{2} (-g) t^2$$

$$\Delta y = (20.0 \text{ m/s})(\sin 40.0^\circ)(1.63 \text{ s}) + \frac{1}{2} (-9.80 \text{ m/s}^2)(1.63 \text{ s})^2$$

$$\Delta y = 7.96 \text{ m}$$

(c) Calculate the angle that the stream of water makes with the building just before it strikes it. (3 marks)

$$v_{fx} = v_{ix} = v_i \cos \theta_i = (20.0 \text{ m/s}) \cos(40.0^\circ)$$

$$v_{fx} = 15.3 \text{ m/s}$$

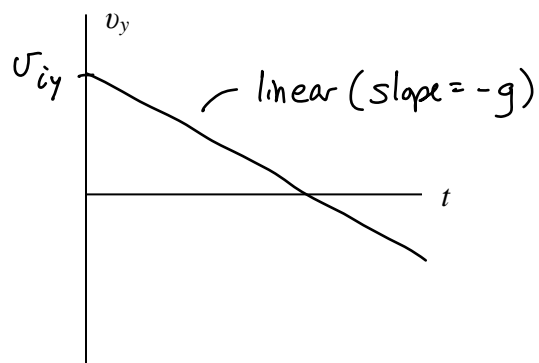
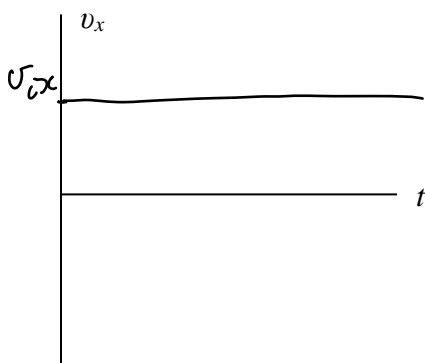
11.5° below horizontal

$$v_{fy} = v_{iy} + a_y t = v_i \sin \theta_i - g t = (20.0 \text{ m/s}) \sin(40.0^\circ) - (9.80 \text{ m/s}^2)(1.63 \text{ s})$$

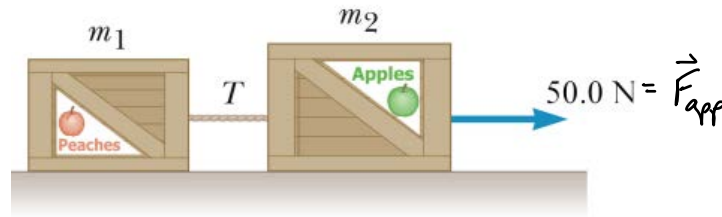
$$v_{fy} = -3.12 \text{ m/s}$$

$$\therefore \theta_f = \text{invtan} \left(\left| \frac{v_{fy}}{v_{fx}} \right| \right) = \text{invtan} \left(\left| \frac{-3.12 \text{ m/s}}{15.3 \text{ m/s}} \right| \right) = 11.5^\circ$$

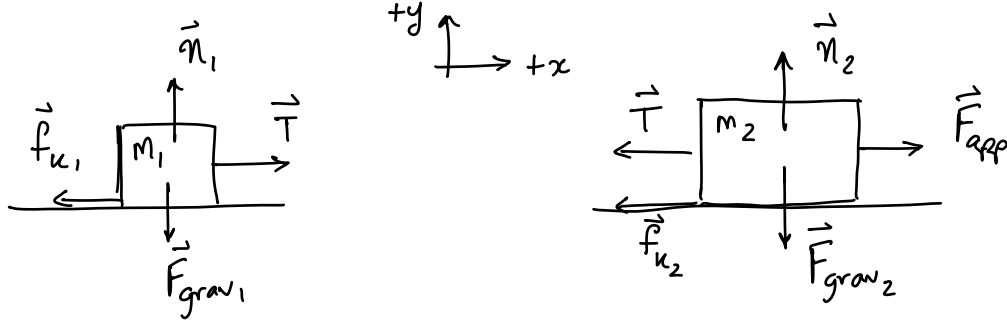
(d) Draw the component-of-velocity versus time graphs from the moment that the stream of water leaves the hose until it reaches the building. Labeling is not required. (The $+x$ direction is horizontal to the right and the $+y$ direction is vertically upward.) (2 marks)



B3. Two boxes of fruit on a horizontal surface are connected by a light, horizontal rope as shown in the figure. $m_1 = 10.0 \text{ kg}$ and $m_2 = 20.0 \text{ kg}$. A force of 50.0 N is applied to the 20.0-kg box. The coefficient of kinetic friction between the boxes and the surface is 0.100 .



(a) Draw a free-body diagram for each box and show the coordinate system. (3 marks)



(b) Calculate the magnitude of the acceleration of each box. (4 marks)

For each box, $\sum \vec{F}_y = 0$. Boxes are connected and move together, $\therefore \vec{a}_1 = \vec{a}_2$.
 $\therefore +n_1 - F_{\text{grav}_1} = 0$ and similarly, $n_2 = m_2 g$ 0.687 m/s^2
 $n_1 = m_1 g$

$$\sum \vec{F}_{x_1} = m_1 \vec{a} \Rightarrow T - f_{k_1} = m_1 a \Rightarrow T = m_1 a + f_{k_1} = m_1 a + \mu_k n_1$$

$$T = m_1 a + \mu_k m_1 g \quad \textcircled{1}$$

$$\sum \vec{F}_{x_2} = m_2 \vec{a} \Rightarrow F_{\text{app}} - T - f_{k_2} = m_2 a \Rightarrow F_{\text{app}} - (m_1 a + \mu_k m_1 g) - \mu_k m_2 g = m_2 a$$

$$F_{\text{app}} - m_1 a - \mu_k m_1 g - \mu_k m_2 g = m_2 a$$

$$F_{\text{app}} - \mu_k m_1 g - \mu_k m_2 g = m_2 a + m_1 a$$

$$a = \frac{F_{\text{app}} - \mu_k g (m_1 + m_2)}{m_2 + m_1} = \frac{50.0 \text{ N} - (0.100)(9.80 \text{ m/s}^2)(10.0 \text{ kg} + 20.0 \text{ kg})}{(20.0 \text{ kg} + 10.0 \text{ kg})}$$

$$\boxed{a = 0.687 \text{ m/s}^2}$$

(c) Calculate the magnitude of the tension T in the rope. If you did not obtain an answer for (b), use a value of 0.700 m/s^2 . (3 marks)

From $\textcircled{1}$ above,

$$\boxed{16.7 \text{ N}}$$

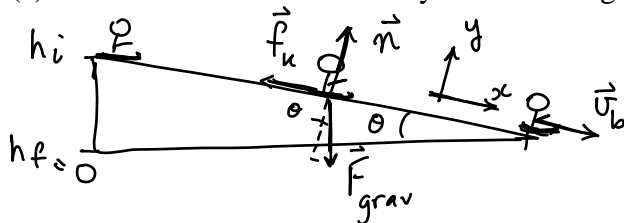
$$T = m_1 a + \mu_k m_1 g$$

$$T = (10.0 \text{ kg})(0.687 \text{ m/s}^2) + (0.100)(10.0 \text{ kg})(9.80 \text{ m/s}^2)$$

$$\boxed{T = 16.7 \text{ N}}$$

B4. A skier starts from rest at the top of a hill that is inclined at 9.50° with respect to the horizontal. The hillside is 195 m long, and the coefficient of kinetic friction between snow and skis is 0.0750. The total mass of the skier and his equipment is 75.0 kg.

(a) Calculate the work done by friction along the 195-m hillside. (4 marks)



$$-1.06 \times 10^4 \text{ J}$$

$$\sum F_y = 0$$

$$+n - F_{\text{grav}} \cos \theta = 0 \Rightarrow n = mg \cos \theta$$

$$W_{fr} = (f_k \cos 180^\circ) d$$

$$W_{fr} = \mu_k n (-1) d = -\mu_k mg \cos \theta d$$

$$W_{fr} = - (0.0750)(75.0 \text{ kg})(9.80 \text{ m/s}^2)(\cos(9.50^\circ))195 \text{ m} = -1.06 \times 10^4 \text{ J}$$

(b) Calculate the speed of the skier at the bottom of the hill. If you did not obtain an answer for (a), use a value of $-1.10 \times 10^4 \text{ J}$. (3 marks)

$$E_i + W_{nc} = E_b$$

$$18.7 \text{ m/s}$$

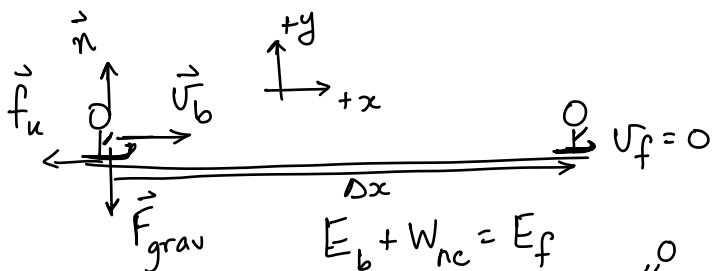
$$KE_i + PE_i + W_{fr} = KE_b + PE_b$$

$$0 + mgh_i + W_{fr} = \frac{1}{2}m v_b^2 + 0 \Rightarrow \frac{2}{m}(mgh_i + W_{fr}) = v_b^2 ; h_i = d \sin \theta$$

$$v_b = \left[\frac{2}{75.0 \text{ kg}} \left((75.0 \text{ kg})(9.80 \text{ m/s}^2)(195 \text{ m}) \sin(9.50^\circ) - 1.06 \times 10^4 \text{ J} \right) \right]^{1/2}$$

$$v_b = 18.659 \text{ m/s} = 18.7 \text{ m/s}$$

(c) At the bottom of the hill, the snow is level and the coefficient of kinetic friction is unchanged. Calculate the distance that the skier glides along the horizontal portion of the snow before coming to rest. If you did not obtain an answer for (b), use a value of 19.0 m/s. (3 marks)



$$237 \text{ m}$$

$$\sum F_y = 0 \Rightarrow +n - F_{\text{grav}} = 0$$

$$n = F_{\text{grav}}$$

$$n = mg$$

$$E_b + W_{nc} = E_f$$

$$KE_b + PE_b + W_{fr} = KE_f + PE_f$$

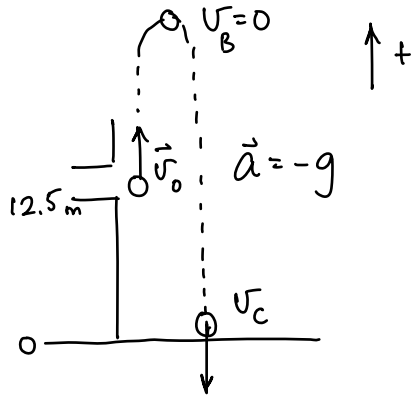
$$\frac{1}{2}m v_b^2 + W_{fr} = 0 \Rightarrow \frac{1}{2}m v_b^2 + \mu_k mg (\cos 180^\circ) (\Delta x) = 0$$

$$\frac{1}{2}m v_b^2 - \mu_k mg \Delta x = 0 \Rightarrow \Delta x = \frac{\frac{1}{2}m v_b^2}{\mu_k mg} = \frac{\frac{1}{2}(18.659 \text{ m/s})^2}{(0.0750)(9.80 \text{ m/s}^2)} = 237 \text{ m}$$

END OF EXAMINATION

B1. While leaning out of a window 12.5 m above the ground, you throw a ball vertically-upward with an initial speed of 25.0 m/s. You may assume that the ball is in free-fall while in the air.

(a) Calculate the maximum height that the ball reaches, relative to the window. (3 marks)



$$v_0 = +25.0 \text{ m/s}$$

At max. height, $v_B = 0$

$$v_B^2 = v_0^2 + 2(-g)(\Delta y)$$

$$0 = v_0^2 - 2g\Delta y \Rightarrow \Delta y = \frac{v_0^2}{2g}$$

$$\Delta y = \frac{(25.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 31.9 \text{ m}$$

31.9 m

(b) Calculate the speed of the ball just before it hits the ground. (3 marks)

$$v_c^2 = v_0^2 + 2a\Delta y_{\text{tot}}$$

$$v_c^2 = v_0^2 + 2(-g)(\Delta y_{\text{tot}})$$

$$v_c = [(25.0 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(-12.5 \text{ m})]^{1/2} = 29.5 \text{ m/s}$$

29.5 m/s

(c) Calculate the time that the ball was in the air (from just after release until just before hitting the ground). (4 marks)

$$\Delta y = v_0 t + \frac{1}{2} a t^2 \Rightarrow 0 = v_0 t + \frac{1}{2} (-g) t^2 - \Delta y$$

$$\left(\frac{1}{2} g\right) t^2 - v_0 t + \Delta y = 0$$

$$t = \frac{-(-v_0) \pm \sqrt{(-v_0)^2 - 4\left(\frac{1}{2} g\right)(\Delta y)}}{2\left(\frac{1}{2} g\right)}$$

$$t = \frac{-(-25.0 \text{ m/s}) \pm \sqrt{(-25.0 \text{ m/s})^2 - 4(4.90 \text{ m/s}^2)(-12.5 \text{ m})}}{9.80 \text{ m/s}^2}$$

$$t = 5.56 \text{ s} \text{ or } -0.459 \text{ s}$$

ALT. SOLUTION:

$$v_c = v_0 + at$$

$$t = \frac{v_c - v_0}{a}$$

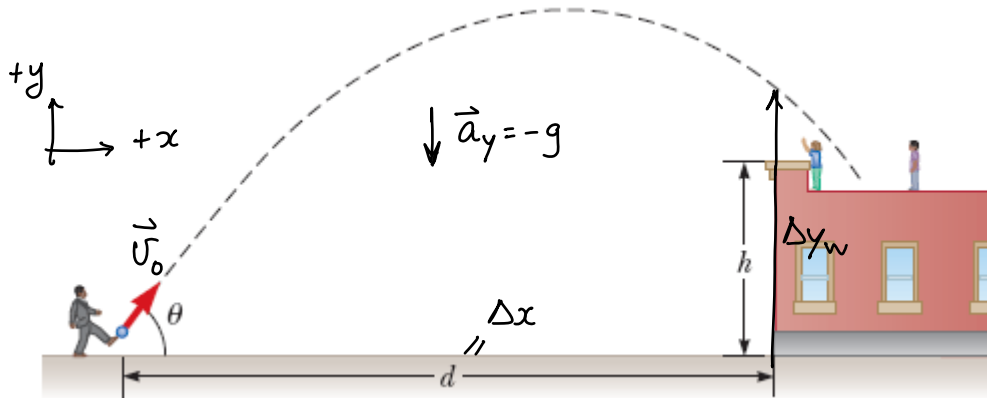
$$t = \frac{v_c - v_0}{-g}$$

$$t = \frac{-29.5 \text{ m/s} - 25.0 \text{ m/s}}{-9.80 \text{ m/s}^2}$$

$$t = 5.56 \text{ s}$$

5.56 s

B2. A children's playground is located on top of a building. The playground is 6.00 m above street level, and is surrounded by a 1.00 m high wall that makes the sides of the building 7.00 m high. A person walking by the building finds a ball on the street and kicks it back to the children on the playground. The person kicks the ball a horizontal distance $d = 24.0$ m away from the building at an angle $\theta = 53.0^\circ$ above the horizontal. It takes the ball 2.20 s to cover the distance d (and reach a point vertically above the front edge of the wall).



(a) Calculate the speed with which the ball was launched. (3 marks)

$$\Delta x = v_{0x} t \Rightarrow d = v_0 \cos \theta \cdot t$$

$$v_0 = \frac{d}{(\cos \theta) t} = \frac{24.0 \text{ m}}{(\cos 53.0^\circ)(2.20 \text{ s})}$$

18.1 m/s

$$v_0 = 18.127 \text{ m/s} = \boxed{18.1 \text{ m/s}}$$

(b) Calculate the vertical distance by which the ball clears the 7.00 m-high wall. If you did not obtain an answer for (a), use a value of 19.0 m/s. (4 marks)

$$\Delta y_w = v_{0y} t + \frac{1}{2} a_y t^2$$

1.13 m

$$\Delta y_w = (v_0 \sin \theta) t + \frac{1}{2} (-g) t^2$$

$$\Delta y_w = (18.127 \frac{\text{m}}{\text{s}})(\sin 53.0^\circ)(2.20 \text{ s}) - \frac{1}{2} (9.80 \text{ m/s}^2)(2.20 \text{ s})^2$$

$$\Delta y_w = 8.13 \text{ m}$$

$$\therefore \text{height above wall} = 8.13 \text{ m} - 7.00 \text{ m} = \boxed{1.13 \text{ m}}$$

(c) Calculate the total time that the ball is in the air. (3 marks)

$$\Delta y_{\text{tot}} = +6.00 \text{ m}$$

$$v_{0y} = v_0 \sin \theta = (18.127 \text{ m/s})(\sin 53.0^\circ)$$

2.46 s

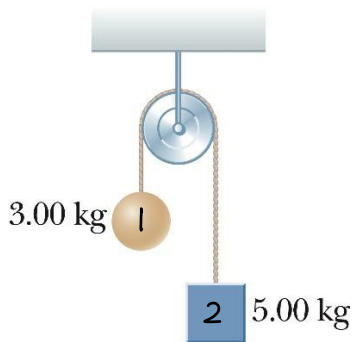
$$v_{0y} = +14.48 \text{ m/s}$$

$$\Delta y_{\text{tot}} = v_{0y} t_{\text{tot}} + \frac{1}{2} a_y t_{\text{tot}}^2 \Rightarrow 0 = \frac{1}{2} (-g) t_{\text{tot}}^2 + v_{0y} t_{\text{tot}} - \Delta y_{\text{tot}}$$

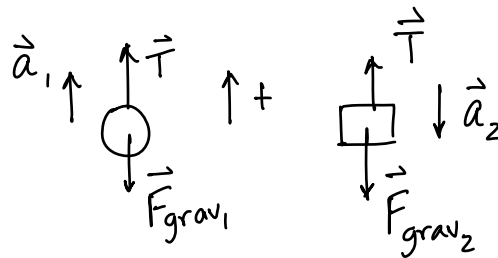
$$t_{\text{tot}} = \frac{-v_{0y} \pm \sqrt{v_{0y}^2 - 4(-\frac{1}{2}g)(-\Delta y_{\text{tot}})}}{2(\frac{1}{2}(-g))} = \frac{-14.48 \text{ m/s} \pm \sqrt{(14.48 \text{ m/s})^2 - 4(4.90 \frac{\text{m}}{\text{s}^2})(6.00 \text{ m})}}{-9.80 \text{ m/s}^2}$$

$$t_{\text{tot}} = \boxed{2.46 \text{ s}}; 0.498 \text{ s}$$

B3. Two objects with masses of $m_1 = 3.00 \text{ kg}$ and $m_2 = 5.00 \text{ kg}$ are connected by a light string that passes over a frictionless pulley as shown in the figure.



(a) Draw the free-body diagrams for each of the masses and show the coordinate system. (3 marks)



(b) Calculate the magnitude of the tension in the string. (4 marks)

Apply Newton II to each object:

$$\sum \vec{F}_1 = m_1 \vec{a}_1 \quad \sum \vec{F}_2 = m_2 \vec{a}_2$$

$$+T - F_{\text{grav}_1} = m_1 a_1 \quad +T - F_{\text{grav}_2} = m_2 (-a_2)$$

$$+T - m_1 g = m_1 a_1 \quad +T - m_2 g = -m_2 a_2$$

Note that $a_1 = a_2 = a$

$$\therefore T - m_1 g = m_1 a$$

$$a = \frac{T}{m_1} - g$$

$$T - m_2 g = -m_2 \left(\frac{T}{m_1} - g \right)$$

$$T - m_2 g = -\frac{m_2}{m_1} T + m_2 g$$

$$T \left(1 + \frac{m_2}{m_1} \right) = 2 m_2 g$$

$$T = \left(\frac{2 m_2}{1 + \frac{m_2}{m_1}} \right) g$$

$$T = \left(\frac{2(5.00 \text{ kg})}{1 + \frac{5.00 \text{ kg}}{3.00 \text{ kg}}} \right) g$$

$$T = (3.75)(9.80 \text{ m/s}^2)$$

36.8 N

(c) Calculate the magnitude of the acceleration of each mass. If you did not obtain an answer (for 36.8 N) (b), use a value of 37.5 N. (3 marks)

From above, $a = \frac{T}{m_1} - g$

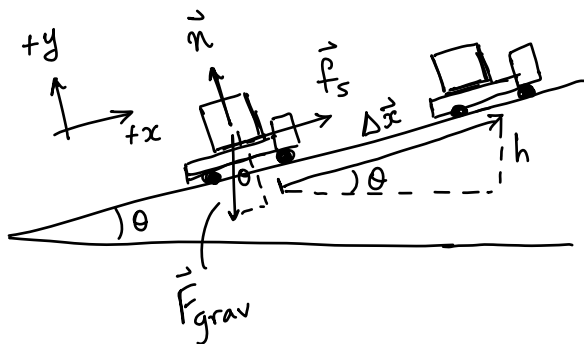
2.45 m/s²

$$a = \frac{36.75 \text{ N}}{3.00 \text{ kg}} - 9.80 \text{ m/s}^2$$

$a = 2.45 \text{ m/s}^2$

B4. A truck travels uphill with constant velocity on a highway with a 7.00° slope. A 50.0-kg package sits on the floor of the back of the truck and does not slide, due to a static frictional force.

(a) Draw a free-body diagram for the package and show the coordinate system. (2 marks)



$$\begin{aligned} \sum F_x &= 0 \\ +f_s - F_{\text{grav}} \sin \theta &= 0 \\ f_s &= mg \sin \theta \end{aligned}$$

(b) During an interval in which the truck travels 341 m , calculate the net work done on the package. (2 marks)

The package is moving at constant speed. $\therefore \Delta KE = 0$, so from the Work-Energy Theorem, $W_{\text{net}} = 0$.

0

(c) During an interval in which the truck travels 341 m , calculate the work done on the package by the force of gravity. (2 marks)

$$W_{\text{grav}} = F_{\text{grav}} \cos(\theta + 90^\circ) \Delta x$$

$$W_{\text{grav}} = mg \cos(97.0^\circ) \Delta x$$

$$W_{\text{grav}} = (50.0\text{ kg})(9.80\text{ m/s}^2)(\cos(97.0^\circ))341\text{ m}$$

$$W_{\text{grav}} = -2.04 \times 10^4\text{ J}$$

$-2.04 \times 10^4\text{ J}$

ALT. SOLUTION:

$$W_{\text{grav}} = -\Delta PE_{\text{grav}}$$

$$W_{\text{grav}} = -mgh = -mg \Delta x \sin \theta$$

$$W_{\text{grav}} = -2.04 \times 10^4\text{ J}$$

(d) During an interval in which the truck travels 341 m , calculate the work done on the package by the normal force. (2 marks)

$$\text{Since } \vec{n} \perp \Delta \vec{x}, W_n = 0$$

0

(e) During an interval in which the truck travels 341 m , calculate the work done on the package by the static frictional force. (2 marks)

$$W_{\text{fr}} = f_s \cos 0^\circ \Delta x; f_s = mg \sin \theta \text{ (see (a))}$$

$$W_{\text{fr}} = (mg \sin \theta)(\cos 0^\circ) \Delta x$$

$$W_{\text{fr}} = (50.0\text{ kg})(9.80\text{ m/s}^2)(\sin 7.00^\circ)(341\text{ m})$$

$$W_{\text{fr}} = 2.04 \times 10^4\text{ J}$$

$2.04 \times 10^4\text{ J}$

ALT. SOLUTION:

$$W_{\text{net}} = 0$$

$$W_{\text{grav}} + W_{\text{fr}} + W_n = 0$$

$$W_{\text{fr}} = -W_n - W_{\text{grav}}$$

$$W_{\text{fr}} = 0 - (-2.04 \times 10^4\text{ J})$$

$$W_{\text{fr}} = 2.04 \times 10^4\text{ J}$$

END OF EXAMINATION