UNIVERSITY OF SASKATCHEWAN

Department of Physics and Engineering Physics

Physics 117.3
<u>MIDTERM TEST</u>

February 12, 2015 Time: 90 minutes

NAME: SOLUTIONS STUDENT NO.: ____________

(Last) **Please Print** (Given)

LECTURE SECTION (please check):

- ❑ 01 Dr. Y. Yao
- ❑ 02 B. Zulkoskey
- ❑ C16 Dr. A. Farahani

<u>INSTRUCTIONS:</u>

1. This is a closed book exam.
2. The test package includes a test paper (this document), a formula sheet, and an OMR sheet. The test paper consists of 8 pages, including this cover page. **It is the responsibility of the student to check that the test paper is complete**.
3. Only a basic scientific calculator (e.g. Texas Instruments TI-30X series, Hewlett-Packard HP 10s or 30S) may be used. Graphing or programmable calculators, or calculators with communication capability, are **<u>not</u>** allowed.
4. Enter your name and student number on the cover of the test paper and check the appropriate box for your lecture section. Also enter your student number in the top right-hand corner of each page of the test paper.
5. Enter your name and NSID on the OMR sheet.
6. The test paper, the formula sheet and the OMR sheet must all be submitted.
7. The marked test paper will be returned. The formula sheet and the OMR sheet will <u>NOT</u> be returned.

ONLY THE <u>THREE</u> PART B QUESTIONS THAT <u>YOU INDICATE</u> WILL BE MARKED

PLEASE <u>INDICATE</u> WHICH <u>THREE</u> PART B QUESTIONS ARE TO BE MARKED

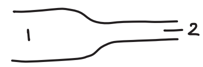
QUESTION NUMBER	TO BE MARKED	MAXIMUM MARKS	MARKS OBTAINED
A1-15	☑	15	
B1	❑	10	
B2	❑	10	
B3	❑	10	
B4	❑	10	
TOTAL		45	

continued on page 2...

PART A

FOR EACH OF THE FOLLOWING QUESTIONS IN PART A, ENTER THE MOST APPROPRIATE RESPONSE ON THE OMR SHEET.

A1. An ideal incompressible fluid is flowing through a horizontal pipe with a constriction. One end of the pipe has a radius of R and the other end of the pipe has a radius of ½ R. Which one of the following statements is true?



E

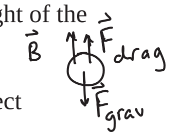
(A) Both the flow speed and pressure are higher at the larger end.
(B) The flow speed is the same throughout the pipe but the pressure is lower at the larger end.
(C) The flow speed at the larger end is half the flow speed at the narrower end.
(D) The pressure is the same throughout the pipe, but the flow speed at the larger end is four times the flow speed at the narrower end.
(E) The pressure is lower at the narrower end.

Continuity Eq'n $\Rightarrow A_1 v_1 = A_2 v_2 \Rightarrow v_2 > v_1$
$\therefore$ Bernoulli's, $P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$
$\Rightarrow P_1 > P_2$
$(y_1 = y_2)$

A2. Which one of the following statements best explains why an object released from rest in a viscous fluid reaches a terminal speed? (The density of the object is greater than the density of the fluid.)

E

(A) The magnitude of the buoyant force on the object is greater than the magnitude of the weight of the object.
(B) The magnitude of the buoyant force on the object equals the magnitude of the weight of the object.
(C) The resistive drag force of the fluid on the object is constant.
(D) The resistive drag force of the fluid on the object decreases as the speed of the object increases.
(E) The resistive drag force of the fluid on the object increases as the speed of the object increases.



Initially $F_{drag} = 0$ so object accelerates downward. As speed increases so does F_{drag}, until equilibrium is reached $\therefore$ speed becomes constant

A3. Blood with viscosity η is flowing through a vein of radius R_1 and length L, with a volume flow rate of Q_1. The vein contracts, so that the new radius R_2 is 85% of the original radius. If the pressure difference between the two ends of the vein remains the same, what is the new volume flow rate Q_2 in terms of Q_1?

D

Poiseville's Law: $Q = \frac{\Delta V}{\Delta t} = \frac{\pi R^4 (P_1 - P_2)}{8\eta L} \Rightarrow Q \propto R^4$

(A) $Q_2 = 0.85 Q_1$ (B) $Q_2 = 0.72 Q_1$ (C) $Q_2 = \frac{Q_2}{0.85} Q_1$ (D) $Q_2 = 0.52 Q_1$ (E) $Q_2 = 0.61 Q_1$

$\therefore \; Q_1 / Q_2 = R_1^4 / R_2^4 \Rightarrow Q_2 = \left(R_2 / R_1\right)^4 \cdot Q_1 = (0.85)^4 Q_1 = 0.52 Q_1$

A4. Two hoses, one of 20-mm diameter, the other of 15-mm diameter, are connected to a faucet, one after the other. At the open end of the hose, the flow of water is 10 litres per minute. Through which hose does the water flow faster?

B

Continuity Eq'n: $Q = \frac{\Delta V}{\Delta t} = A_1 v_1 = A_2 v_2$

(A) the 20-mm hose
(B) the 15-mm hose
(C) The flow speed is the same in both hoses.
(D) The answer depends on which of the hoses comes first in the flow.
(E) The answer depends on the lengths of the hoses.

$\therefore$ flow speed is greater in hose with smaller cross-sectional area.

A5. A mass is hung on a spring (k = 100 N/m) and set in simple harmonic motion. The period of oscillation is T. If the spring is cut in half, so that now k = 200 N/m, and the same mass is set in simple harmonic motion again, what is the new period of oscillation?

D

$T = 2\pi\sqrt{\frac{m}{k}}$

(A) T (B) $\frac{1}{2} T$ (C) $\sqrt{2}\, T$ (D) $\frac{1}{\sqrt{2}} T$ (E) $2T$

$T \propto \frac{1}{\sqrt{k}}$; $T_2 / T_1 = \sqrt{k_1 / k_2} \Rightarrow T_2 = \sqrt{\frac{k}{2k}} \cdot T = \frac{1}{\sqrt{2}} \cdot T$

A6. If one could transport a simple pendulum of fixed length from the Earth's surface to the Moon's, where the acceleration due to gravity is one-sixth (1/6) that on the Earth, how would the pendulum's frequency of oscillation on the Moon compare to its frequency of oscillation on the Earth?

E

(A) $f_{\text{Moon}} = 6\, f_{\text{Earth}}$ (B) $f_{\text{Moon}} = 2.5\, f_{\text{Earth}}$ (C) $f_{\text{Moon}} = 0.86\, f_{\text{Earth}}$
(D) $f_{\text{Moon}} = 0.17\, f_{\text{Earth}}$ (E) $f_{\text{Moon}} = 0.41\, f_{\text{Earth}}$

$T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \Rightarrow f \propto \sqrt{g}$; $\frac{f_{moon}}{f_{Earth}} = \sqrt{\frac{g_{moon}}{g_{Earth}}}$

$f_{moon} = \sqrt{\frac{1}{6}} \cdot f_{Earth} = 0.41 f_{Earth}$

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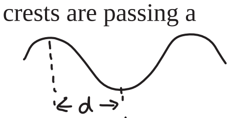
C A7. The distance between the crest of a wave and the next trough is 1 m. If wave crests are passing a particular point at the rate of 2 per second, what is the speed of the wave?



f "

(A) 1 m/s (B) 2 m/s (C) 4 m/s (D) 8 m/s
(E) The wave speed is impossible to determine from the given information.

$1\,m = \frac{1}{2}\lambda \Rightarrow \lambda = 2m$

$v = f\lambda$

$v = 4\,m/s$

D A8. An object attached to a spring is moving with simple harmonic motion of amplitude *A*. When the kinetic energy of the object equals the potential energy stored in the spring, what is the position of the object relative to the equilibrium position?

(A) A (B) 0 (C) $\frac{1}{2}A$ (D) $\frac{1}{\sqrt{2}}A$ (E) $2A$

$E = KE + PE \Rightarrow \frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$. When $KE = PE$, then $\frac{1}{2}kA^2 = 2\left(\frac{1}{2}kx^2\right) \Rightarrow A^2 = 2x^2$

$x = \frac{1}{\sqrt{2}} \cdot A$

E A9. As you travel down the highway in your car, an ambulance moves away from you at a high speed, sounding its siren at a frequency of 400 Hz. Which one of the following statements is true? source moving away from you, you hear a lower frequency (Doppler Effect)

(A) You and the ambulance driver both hear a frequency greater than 400 Hz.
(B) You and the ambulance driver both hear a frequency less than 400 Hz.
(C) You and the ambulance driver both hear a frequency of 400 Hz.
(D) You hear a frequency greater than 400 Hz, whereas the ambulance driver hears a frequency of 400 Hz.
(E) You hear a frequency less than 400 Hz, whereas the ambulance driver hears a frequency of 400 Hz.

Ambulance driver is not moving relative to siren.

C A10. A point source broadcasts sound into a uniform medium. An observer moves away from the source at a certain speed. If the power is increased by a factor of 4 and the distance from the source is doubled, what is the resulting change in decibel level?

$I = \frac{P}{A} = \frac{P}{4\pi r^2}$

(A) The decibel level goes down by more than 10 dB.
(B) The decibel level goes up by less than 10 dB.
(C) There is no change in the decibel level.
(D) The answer cannot be determined because the speed of the observer is not known.
(E) The decibel level goes down by less than 10 dB.

$P_2 = 4P_1$; $r_2 = 2r_1 \Rightarrow I_2 = I_1$

$\Rightarrow \beta_2 = \beta_1$

A A11. A sound wave travelling in air has a frequency f and wavelength λ. A second sound wave travelling in air has a wavelength of $\lambda/4$. What is the frequency of the second sound wave?

(A) $4f$ (B) $2f$ (C) f (D) $\frac{1}{2}f$ (E) $\frac{1}{4}f$

speed of sound doesn't change $\Rightarrow f_1\lambda_1 = f_2\lambda_2 \Rightarrow f_2 = f\lambda / \lambda/4 = 4f$

B A12. Tripling the power output from a speaker emitting a single frequency of sound will result in what increase in intensity level? $P_2 = 3P_1 \Rightarrow I_2 = 3I_1 \Rightarrow \beta_2 - \beta_1 = 10\log_{10}(I_2/I_1)$

(A) 0.33 dB (B) 4.8 dB (C) 3.0 dB (D) 9.0 dB (E) 6.2 dB

$= 10\log(3) = 4.8\,dB$

C A13. Choose the option that correctly completes this sentence: "When two waves are out of phase by 540°, destructive interference will occur". destructive interference $\Rightarrow$ out of phase by

(A) 90° (B) 270° (C) 540° (D) 720° (E) 450°

$180° + n(360°)$
where n is an integer

C A14. Which one of the following sets of resonant frequencies **cannot** apply to a pipe that is open at one end and closed at the other? open/closed $\Rightarrow f_{res} = f_1, 3f_1, 5f_1, 7f_1 \ldots$

(A) 100 Hz, 300 Hz, 500 Hz, and no other frequencies between 100 Hz and 500 Hz. T
(B) 20 Hz, 60 Hz, 100 Hz, and no other frequencies between 20 Hz and 100 Hz. T
→(C) 100 Hz, 200 Hz, 300 Hz, and no other frequencies between 100 Hz and 300 Hz. F
(D) 50 Hz, 150 Hz, 250 Hz, and no other frequencies between 100 Hz and 1600 Hz. T
(E) 30 Hz, 90 Hz, 150 Hz, and no other frequencies between 30 Hz and 150 Hz. T

E A15. When two tuning forks are sounded at the same time, a beat frequency of 5 Hz occurs. If one of the tuning forks has a frequency of 245 Hz, what is the frequency of the other tuning fork?

(A) 235 Hz (B) 242.5 Hz (C) 247.5 Hz
(D) 240 Hz is the only possibility (E) either 240 Hz or 250 Hz

$f_{beat} = |f_2 - f_1|$

continued on page 4...

PART B

ANSWER THREE OF THE PART B QUESTIONS ON THE FOLLOWING PAGES AND INDICATE YOUR CHOICES ON THE COVER PAGE.

FOR EACH OF YOUR CHOSEN PART B QUESTIONS ON THE FOLLOWING PAGES, GIVE THE COMPLETE SOLUTION AND ENTER THE FINAL ANSWER IN THE BOX PROVIDED.

THE ANSWERS MUST CONTAIN THREE SIGNIFICANT FIGURES AND THE UNITS MUST BE GIVEN.

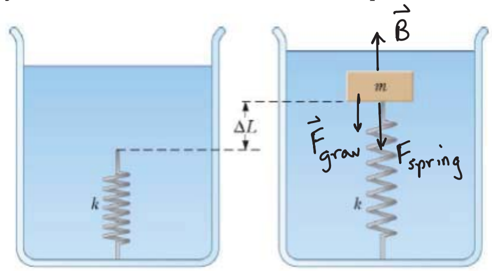
SHOW AND EXPLAIN YOUR WORK – NO CREDIT WILL BE GIVEN FOR ANSWERS ONLY.

EQUATIONS NOT PROVIDED ON THE FORMULAE SHEET MUST BE DERIVED.

USE THE BACK OF THE PREVIOUS PAGE FOR YOUR ROUGH WORK.

continued on page 5...

B1. A light spring of force constant $k = 1.00 \times 10^2$ N/m is attached to the bottom of a large beaker of water as shown. A 4.00-kg block of wood (density = 6.50×10^2 kg/m^3) is connected to the spring, and the block-spring system is allowed to come to static equilibrium.



$\uparrow +$

(a) Calculate the buoyant force exerted by the water on the block. (3 marks)

$\boxed{60.3 \text{ N}}$

$B = \rho_{fluid} V_{fluid}\, g$

completely submerged $\Rightarrow V_{fluid} = V_{object}$; $\rho = \frac{M}{V} \Rightarrow V_{object} = \frac{M_{object}}{\rho_{object}}$

$B = \rho_{fluid}\left(\frac{M_{object}}{\rho_{object}}\right) g$

$$B = (1.00 \times 10^3 \text{ kg/m}^3)\left(\frac{4.00 \text{ kg}}{6.50 \times 10^2 \text{ kg/m}^3}\right)(9.80 \text{ m/s}^2) = 60.3 \text{ N}$$

(b) Calculate the elongation, ΔL, of the spring. If you did not obtain an answer for (a), use a value of 49.5 N. (4 marks)

$\boxed{0.211 \text{ m}}$

At equilibrium, $\Sigma \vec{F} = 0$

$+B - F_{spring} - F_{grav} = 0$

$F_{spring} = B - F_{grav}$

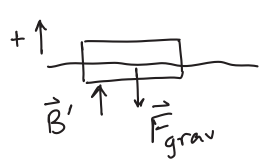
$k(\Delta L) = B - F_{grav} \Rightarrow \Delta L = \frac{B - F_{grav}}{k} = \frac{B - M_{object}\, g}{k}$

$$\Delta L = \frac{60.3 \text{ N} - (4.00 \text{ kg})(9.80 \text{ m/s}^2)}{1.00 \times 10^2 \text{ N/m}} = 0.211 \text{ m}$$

ALT. VALUE: $\Delta L_{ALT} = 0.103$ m

(c) The block detaches from the spring and rises to the surface. Once the block has come to rest, calculate the percentage of its volume that is submerged. (3 marks)

$\boxed{65.0\%}$



At equilibrium, $\Sigma \vec{F} = 0$

$B' - F_{grav} = 0$

$\rho_{fluid} V_{fluid}\, g - M_{object}\, g = 0$; $M_{object} = \rho_{object} V_{object}$

$\rho_{fluid} V_{fluid}\, g - \rho_{object} V_{object}\, g = 0$ and $V_{fluid} = V_{submerged}$

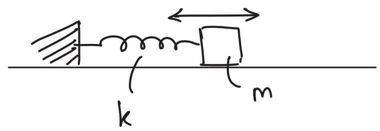
$\therefore \rho_{fluid} V_{submerged} \cancel{g} = \rho_{object} V_{object} \cancel{g} \Rightarrow \frac{V_{submerged}}{V_{object}} = \frac{\rho_{object}}{\rho_{fluid}} = \frac{6.50 \times 10^2 \text{ kg/m}^3}{1.00 \times 10^3 \text{ kg/m}^3}$

$\frac{V_{submerged}}{V_{object}} = 0.650 \Rightarrow 65.0\%$

continued on page 6...

B2. A 326-g object on a horizontal, frictionless surface is attached to a horizontal spring and undergoes simple harmonic motion with a period of 0.250 s. The total energy of the system is 5.83 J.

(a) Calculate the maximum speed of the object. (3 marks)



$E = KE + PE$

$\boxed{5.98\ m/s}$

KE_{max} when $PE = 0$ (equilibrium position)

$E = \frac{1}{2} m v_{max}^2$

$$\therefore\ v_{max} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(5.83\,J)}{0.326\,kg}} = 5.98\ m/s$$

(b) Calculate the force constant of the spring. (4 marks)

$\boxed{206\ kg/s^2}$

$$\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

$$\frac{4\pi^2}{T^2} = \frac{k}{m} \Rightarrow k = \frac{4\pi^2 m}{T^2}$$

$$k = \frac{4\pi^2 (0.326\,kg)}{(0.250\,s)^2} = 206\ kg/s^2$$

(c) Calculate the amplitude of the motion. If you did not obtain an answer for (b), use a value of 201 N/m. (3 marks)

$E = PE_{max} = \frac{1}{2} k A^2$ ($KE = 0$ when $PE = PE_{max}$)

$\boxed{0.238\,m}$

$$\therefore\ A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(5.83\,J)}{206\,kg/s^2}} = 0.238\,m$$

ALT. VALUE: 0.231 m

ALT. METHOD:

SHM, $\therefore\ v_{max} = \omega A$ and $\omega = \sqrt{\frac{k}{m}} \Rightarrow v_{max} = \sqrt{\frac{k}{m}} \cdot A$

$$A = v_{max}\sqrt{\frac{m}{k}} = 5.98\,m/s \sqrt{\frac{0.326\,kg}{206\,kg/s^2}} = 0.238\,m$$

continued on page 7...

B3. A train sounds its horn as it approaches a crossing. The horn can be heard at an intensity level of 50.0 dB by an observer 10.0 km away. Treat the horn as a point source and neglect any absorption of sound by the air. You may assume that the sound energy propagates uniformly in all directions.

(a) Calculate the average rate at which sound energy is generated by the horn. (5 marks)

rate of energy production = power, P

Intensity, $I = \frac{P}{A} \Rightarrow P = I \cdot A = I \cdot 4\pi r^2$

Given intensity level, β: $\beta = 10\,\text{dB} \log_{10}\left(\frac{I}{I_0}\right)$

$$\log_{10}\left(\frac{I}{I_0}\right) = \frac{\beta}{10\,\text{dB}} \Rightarrow 10^{\beta/10\,\text{dB}} = \frac{I}{I_0} \Rightarrow I = \left(10^{\beta/10\,\text{dB}}\right) I_0$$

$$I = \left(10^{50.0\,\text{dB}/10\,\text{dB}}\right)\left(1.00\times10^{-12}\,\text{W/m}^2\right) = \left(1.00\times10^{5}\right)\left(1.00\times10^{-12}\,\text{W/m}^2\right)$$

$$I = 1.00\times10^{-7}\,\text{W/m}^2$$

$$P = I\cdot 4\pi r^2 = \left(1.00\times10^{-7}\,\text{W/m}^2\right)\left(4\pi\left(10.0\times10^{3}\,\text{m}\right)^2\right)$$

$$P = 126\,\text{W}$$

(b) Calculate the intensity level of the horn's sound heard by someone at a distance of 50.0 m from the train. (5 marks)

$\boxed{96.0\,\text{dB}}$

As shown in (a), $\beta_1 = 50.0\,\text{dB}$
$\Rightarrow I_1 = 1.00\times10^{-7}\,\text{W/m}^2$

$I = \frac{P}{A} = \frac{P}{4\pi r^2} \Rightarrow P = I\cdot 4\pi r^2$ For constant power, $P_1 = P_2$

$$\therefore \quad I_1\cdot 4\pi r_1^2 = I_2\cdot 4\pi r_2^2 \Rightarrow I_1 r_1^2 = I_2 r_2^2$$

$I_1 = 1.00\times10^{-7}\,\text{W/m}^2$, $r_1 = 10.0\times10^{3}\,\text{m}$

$r_2 = 50.0\,\text{m}$, $I_2 = ?$

$$I_2 = \frac{I_1 r_1^2}{r_2^2} = \frac{\left(1.00\times10^{-7}\,\text{W/m}^2\right)\left(10.0\times10^{3}\,\text{m}\right)^2}{(50.0\,\text{m})^2} = 4.00\times10^{-3}\,\text{W/m}^2$$

$$\therefore \beta_2 = 10\,\text{dB}\log_{10}\left(\frac{4.00\times10^{-3}\,\text{W/m}^2}{1.00\times10^{-12}\,\text{W/m}^2}\right) = 96.0\,\text{dB}$$

continued on page 8...

B4. A steel wire with mass 25.0 g and length 1.35 m is strung on a bass guitar so that the length of the string that is free to vibrate is 1.10 m.

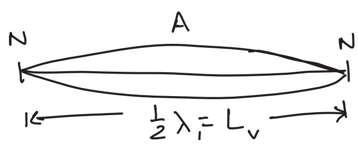
(a) Calculate the linear mass density of the string. (2 marks)

$$\mu = \frac{m}{L} = \frac{25.0 \times 10^{-3}\,\text{kg}}{1.35\,\text{m}}$$

$$\mu = 1.85 \times 10^{-2}\,\text{kg/m}$$

$\boxed{1.85 \times 10^{-2}\,\text{kg/m}}$

(b) Calculate the velocity of the wave on the string that will produce a fundamental frequency of 41.2 Hz. (3 marks)



At fundamental,

$L_v = \frac{1}{2}\lambda_1 \Rightarrow \lambda_1 = 2L$

$\boxed{90.6\,\text{m/s}}$

$$v = f_1\lambda_1 = f_1 \cdot 2L_v = (41.2\,\text{Hz})(2)(1.10\,\text{m}) = 90.6\,\text{m/s}$$

(c) Calculate the tension required to produce a fundamental frequency of 41.2 Hz. (3 marks)

$$v = \sqrt{\frac{F}{\mu}} \Rightarrow v^2 = \frac{F}{\mu} \Rightarrow F = \mu v^2$$

$\boxed{152\,\text{N}}$

$$F = (1.85 \times 10^{-2}\,\text{kg/m})(90.6\,\text{m/s})^2 = 152\,\text{N}$$

(d) Calculate the wavelength of the sound wave produced in air when the string is vibrating at a frequency of 41.2 Hz. The speed of sound in air is 343 m/s. (2 marks)

$$v = f\lambda$$

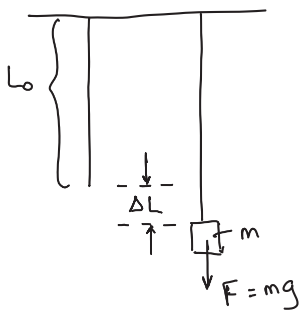
$\boxed{8.33\,\text{m}}$

$$\lambda = \frac{v}{f} = \frac{343\,\text{m/s}}{41.2\,\text{Hz}} = 8.33\,\text{m}$$

END OF EXAMINATION

B1. A rubber band is tied to a hook and hanging vertically from the ceiling. The length of the rubber band is 10.0 cm and its cross-sectional area is 9.00×10^{-6} m^2. The Young's modulus of rubber is 1.00×10^6 Pa.

(a) A mass of 125 g is now attached to the free end of the rubber band and the mass is slowly lowered until it comes to rest at its equilibrium position, at which point the mass is released. Calculate the distance that the rubber band has stretched. (You may assume that the cross-sectional area of the rubber band has not changed.) (4 marks)



Assume that the rubber band behaves elastically.

$\boxed{1.36 \text{ cm}}$

$$\therefore \frac{F}{A} = Y\frac{\Delta L}{L_0} \Rightarrow \Delta L = \frac{L_0 F}{YA} = \frac{L_0 mg}{YA}$$

$$\Delta L = \frac{(0.100\text{m})(0.125\text{kg})(9.80\text{m/s}^2)}{(1.00\times10^6\text{Pa})(9.00\times10^{-6}\text{m}^2)}$$

$$\Delta L = 1.36\times10^{-2}\text{m} = 1.36\text{ cm}$$

(b) Treating the rubber band as an ideal spring, calculate the force constant of the rubber band. If you did not obtain an answer for (a), use a value of 0.0125 m. (3 marks)

Ideal spring, $|F| = k|x|$

$\boxed{90.1 \text{ N/m}}$

In the above diagram, when the mass is at equilibrium, the force exerted by the elastic band is mg, upward

$$\therefore mg = k|\Delta L| \Rightarrow k = \frac{mg}{|\Delta L|}$$

$$k = \frac{(0.125\text{ kg})(9.80\text{m/s}^2)}{0.0136\text{m}}$$

$$k = 90.1 \text{ N/m}$$

ALT. ANSWER: $k = 98.0 \text{ N/m}$

(c) The 125 g mass is now pulled down from its equilibrium position and released. Calculate the frequency of the vertical oscillations of the mass. If you did not obtain an answer for (b), use a value of 92.5 N/m (3 marks).

$$\omega = 2\pi f = \sqrt{\frac{k}{m}}$$

$\boxed{4.27 \text{ Hz}}$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

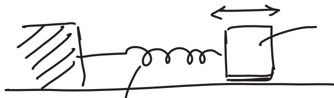
$$f = \frac{1}{2\pi}\sqrt{\frac{90.1\text{ N/m}}{0.125\text{ kg}}} = 4.27\text{ Hz}$$

ALT. ANSWER: 4.33 Hz

continued on page 6...

B2. A 0.400-kg object connected to a light spring with a force constant of 19.6 N/m is on a horizontal frictionless surface. The spring is compressed 4.00 cm from its equilibrium position and the mass is released from rest.

(a) Calculate the total energy of the mass-spring system. (3 marks)



m = 0.400m

k = 19.6 N/m

$\boxed{1.57 \times 10^{-2}\ \mathrm{J}}$

Amplitude of motion, A (max. compression of spring) = 0.0400m

At max. compression, the mass is at rest.

$$\therefore\ E = KE + PE = 0 + \frac{1}{2}kA^2 = \frac{1}{2}(19.6\,\mathrm{N/m})(0.0400\,\mathrm{m})^2$$

$$E = 1.57 \times 10^{-2}\,\mathrm{J}$$

(b) Calculate the maximum speed of the object. If you did not obtain an answer for (a), use a value of 0.0160 J. (3 marks)

$\boxed{0.280\ \mathrm{m/s}}$

max. speed is reached as the object passes through the equilibrium position, where $x = 0$.

Energy is conserved, and $E = KE + PE$, so

$$E = \frac{1}{2}mv_{max}^2 + 0 \Rightarrow v_{max} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(1.57\times10^{-2}\,\mathrm{J})}{0.400\,\mathrm{kg}}} = 0.280\,\mathrm{m/s}$$

$\boxed{\text{ALT. ANSWER: } 0.283\ \mathrm{m/s}}$

(c) Calculate the distance of the object from the equilibrium position when its speed equals one half of the maximum speed. If you did not obtain an answer for (b), use a value of 0.250 m/s. (4 marks)

$\boxed{3.47\ \mathrm{cm}}$

Again, $E = KE + PE$

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \quad \text{and} \quad v = \frac{1}{2}v_{max}$$

$$E = \frac{1}{2}m\left(\frac{1}{2}v_{max}\right)^2 + \frac{1}{2}kx^2 \Rightarrow 2E = m\left(\frac{1}{2}v_{max}\right)^2 + kx^2$$

$$2E = \frac{1}{4}mv_{max}^2 + kx^2 \Rightarrow kx^2 = 2E - \frac{1}{4}mv_{max}^2$$

$$x = \sqrt{\frac{2E - \frac{1}{4}mv_{max}^2}{k}} = \sqrt{\frac{2(1.57\times10^{-2}\,\mathrm{J}) - \frac{1}{4}(0.400\,\mathrm{kg})(0.280\,\mathrm{m/s})^2}{19.6\,\mathrm{N/m}}}$$

$$x = 3.47\times10^{-2}\,\mathrm{m} = 3.47\,\mathrm{cm}$$

$\boxed{\text{ALT. ANSWER 1: } 3.58\ \mathrm{cm}}$ $v_{max} = 0.250\ \mathrm{m/s}$

$\boxed{\text{ALT. ANSWER 2: } 3.62\ \mathrm{cm}}$ $v_{max} = 0.250\ \mathrm{m/s}$, $E = 0.0160\ \mathrm{J}$

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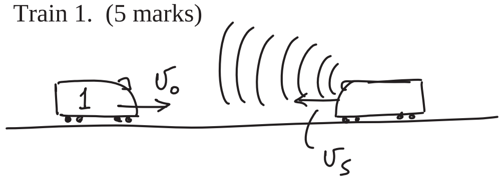
B3. Two trains on separate tracks move toward each other. Train 1 has a speed of 141 km/h; Train 2 has a speed of 102 km/h. Train 2 blows its horn, emitting 995 W sound waves at a frequency of 605 Hz. The speed of sound in air is 343 m/s.

(a) Calculate the frequency of the sound from Train 2's horn that is heard by the engineer of Train 1. (5 marks)



$\boxed{735\ \text{Hz}}$

Both source and observer are moving.

$$f_o = f_s\left(\frac{v + v_o}{v - v_s}\right)$$

observer moving toward source: v_o +ve
source moving toward observer: v_s +ve

Convert train speeds from km/h to m/s.

$$v_o = 141\,\frac{\text{km}}{\text{h}} \times \frac{1000\,\text{m}}{\text{km}} \times \frac{1\,\text{h}}{3600\,\text{s}} = 39.17\,\text{m/s}$$

$$v_s = 102\,\frac{\text{km}}{\text{h}} \times \frac{1000\,\text{m}}{\text{km}} \times \frac{1\,\text{h}}{3600\,\text{s}} = 28.33\,\text{m/s}$$

$$f_o = 605\,\text{Hz}\left(\frac{343\,\text{m/s} + (+39.17\,\text{m/s})}{343\,\text{m/s} - (+28.33\,\text{m/s})}\right) = 735\,\text{Hz}$$

(b) Calculate the intensity level of the sound from Train 2's horn that is heard by the engineer on Train 1 when the two trains are a distance of 10.0 m from each other. You may assume that the sound energy propagates uniformly in all directions. (5 marks)

$\boxed{119\ \text{dB}}$

$$P = 995\,\text{W}$$

$$I = \frac{P}{A} = \frac{P}{4\pi r^2} = \frac{995\,\text{W}}{4\pi(10.0\,\text{m})^2} = 0.792\,\text{W/m}^2$$

$$\beta = 10\,\text{dB}\,\log_{10}\left(\frac{I}{I_o}\right)$$

$$\beta = 10\,\text{dB}\,\log_{10}\left(\frac{0.792\,\text{W/m}^2}{1.00\times10^{-12}\,\text{W/m}^2}\right) = 119\,\text{dB}$$

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B4. A stretched string fixed at each end has a mass of 40.0 g and a length of 8.00 m. The tension in the string is 49.0 N.

(a) Calculate the speed of a wave on the string. (3 marks)

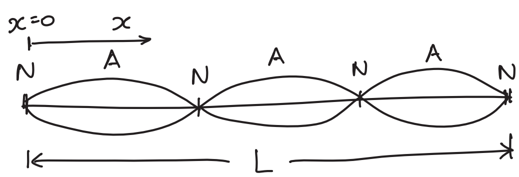
$$v = \sqrt{\frac{F}{\mu}} \; ; \; \mu = \frac{m}{L} \Rightarrow v = \sqrt{\frac{FL}{m}}$$

$$v = \sqrt{\frac{(49.0\,\mathrm{N})(8.00\,\mathrm{m})}{40.0\times 10^{-3}\,\mathrm{kg}}} = 99.0\,\mathrm{m/s}$$

$\boxed{99.0\,\mathrm{m/s}}$

(b) Assume that the string is vibrating at its third harmonic. Choosing the left-hand end of the string to be at $x = 0$, calculate the positions of all the antinodes that occur along the length of the string. (4 marks)

3rd harmonic $\Rightarrow$ 3 $\frac{1}{2}$-λ segments



The nodes occur at $x_N = 0, \frac{1}{3}L, \frac{2}{3}L, L$

$\therefore$ The antinodes occur at $x_A = \frac{1}{6}L, \frac{3}{6}L, \frac{5}{6}L$

$$x_{A_1} = \frac{1}{6}(8.00\,\mathrm{m}) = 1.33\,\mathrm{m}$$
$$x_{A_2} = \frac{3}{6}(8.00\,\mathrm{m}) = 4.00\,\mathrm{m}$$
$$x_{A_3} = \frac{5}{6}(8.00\,\mathrm{m}) = 6.67\,\mathrm{m}$$

1.33 m 4.00 m 6.67 m

(c) Calculate the frequency of vibration when the string is vibrating at its third harmonic. (3 marks)

As shown in the diagram above, at the 3rd harmonic there are 3 ½-λ segments along the length of the wire.

$$L = \frac{3}{2}\lambda_3 \Rightarrow \lambda_3 = \frac{2}{3}L = \frac{2}{3}(8.00\,\mathrm{m}) = 5.33\,\mathrm{m}$$

$$v = f\lambda \Rightarrow f_3 = \frac{v}{\lambda_3} = \frac{99.0\,\mathrm{m/s}}{5.33\,\mathrm{m}} = 18.6\,\mathrm{Hz}$$

$\boxed{18.6\,\mathrm{Hz}}$

END OF EXAMINATION