UNIVERSITY OF SASKATCHEWAN

Department of Physics and Engineering Physics

Physics 117.3

MIDTERM TEST

February 13, 2014 Time: 90 minutes

NAME: Solutions Master STUDENT NO.: ____________

(Last) **Please Print** (Given)

LECTURE SECTION (please check):

- ❑ 01 B. Zulkoskey
- ❑ 02 Dr. J-P St. Maurice
- ❑ C15 F. Dean

INSTRUCTIONS:

1. This is a closed book exam.
2. The test package includes a test paper (this document), a formula sheet, and an OMR sheet. The test paper consists of 8 pages, including this cover page. **It is the responsibility of the student to check that the test paper is complete**.
3. Only a basic scientific calculator (e.g. Texas Instruments TI-30X series, Hewlett-Packard HP 10s or 30S) may be used. Graphing or programmable calculators, or calculators with communication capability, are **not** allowed.
4. Enter your name and student number on the cover of the test paper and check the appropriate box for your lecture section. Also enter your student number in the top right-hand corner of each page of the test paper.
5. Enter your name and NSID on the OMR sheet.
6. The test paper, the formula sheet and the OMR sheet must all be submitted.
7. The marked test paper will be returned. The formula sheet and the OMR sheet will NOT be returned.

ONLY THE THREE PART B QUESTIONS THAT YOU INDICATE WILL BE MARKED

PLEASE INDICATE WHICH THREE PART B QUESTIONS ARE TO BE MARKED

QUESTION NUMBER	TO BE MARKED	MAXIMUM MARKS	MARKS OBTAINED
A1-15	☑	15	
B1	❑	10	
B2	❑	10	
B3	❑	10	
B4	❑	10	
TOTAL		45	

continued on page 2...

PART A

FOR EACH OF THE FOLLOWING QUESTIONS IN PART A, ENTER THE MOST APPROPRIATE RESPONSE ON THE OMR SHEET.

E

A1. Two objects of exactly the same size and shape, one made of wood and the other made of steel, are placed in a container of water. The wood object floats and the steel object sinks to the bottom of the container. Which one of the following statements is **TRUE**? $B = \rho_{fluid} V_{fluid}\, g$

(A) Both objects experience the same magnitude of buoyant force.
(B) The buoyant force on the wood object is directed upward and the buoyant force on the steel object is directed downward.
(C) The buoyant force on both objects is directed downward.
(D) The magnitude of the buoyant force on the wood object is greater than on the steel object.
(E) The magnitude of the buoyant force on the steel object is greater than on the wood object.

The steel object displaces more fluid than the wood object.

C

A2. If the pressure at a depth d below the surface of a lake is $2P_{atm}$ (twice atmospheric pressure), the pressure at a depth $2d$ is

At d, $P = P_{atm} + \rho g d = 2P_{atm}$ $\therefore \rho g d = P_{atm}$

(A) P_{atm}. (B) $2\,P_{atm}$. (C) $3\,P_{atm}$. (D) $4\,P_{atm}$. (E) zero

At $2d$, $P = P_{atm} + \rho g(2d) = P_{atm} + 2\rho g d = P_{atm} + 2(P_{atm})$
$= 3P_{atm}$

P_{atm}, d, $2d$

C

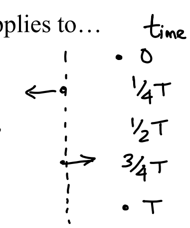
A3. Which one of the following statements regarding Stress and Strain is **FALSE**?

(A) Stress is the force per unit area causing a deformation; Strain is a measure of the amount of the deformation. True
(B) Provided the stress does not exceed the elastic limit of the material, a solid object returns to its original length when the stress is removed. True
(C) A solid object will break as soon as the stress exceeds the elastic limit of the material. False
(D) The maximum stress that a non-ductile object can withstand without breaking is called the ultimate strength. True
(E) If the tensile or compressive stress exceeds the proportional limit then the strain is no longer proportional to the stress. True

D

A4. Choose the phrase that best completes the following sentence: "Bernoulli's equation applies to…

(A) any fluid."
(B) an incompressible fluid, whether viscous or non-viscous."
(C) an incompressible, non-viscous fluid, whether the flow is turbulent or not."
(D) an incompressible, non-viscous fluid in which the flow is non-turbulent."
(E) a static fluid only."



C

A5. An object is in simple harmonic motion with a period of T. If the object was displaced in the positive direction and released at time $t = 0$, which one of the following statements correctly describes the motion of the object at time $t = \frac{3}{4}\,T$?

(A) The object is momentarily at rest at maximum negative displacement.
(B) The object is passing through the equilibrium position, moving in the negative direction.
(C) The object is passing through the equilibrium position, moving in the positive direction.
(D) The object is halfway between the equilibrium position and the position of maximum negative displacement.
(E) The object is halfway between the equilibrium position and the position of maximum positive displacement.

E

A6. Which one of the following statements is **FALSE** regarding a mass-spring system that moves with simple harmonic motion in the absence of friction?

(A) The total energy of the system remains constant. True
(B) The energy of the system is continually transformed between kinetic and potential energy. True
(C) The total energy of the system is proportional to the square of the amplitude. True
(D) The velocity of the oscillating mass has its maximum value when the mass passes through the equilibrium position. True
(E) The potential energy stored in the system is greatest when the mass passes through the equilibrium position. False

$PE = \frac{1}{2}kx^2 \Rightarrow$ max. when $x = A$ (max. displacement)

continued on page 3...

A7. A simple pendulum oscillates with a small amplitude. Its length is doubled and its mass is halved. What happens to the frequency of its motion? (Choose the correct answer.)

A

$T = 2\pi\sqrt{\frac{L}{g}}$

(A) It becomes $1/\sqrt{2}$ as large.
(B) It doubles.
(C) The frequency is unchanged.
(D) It becomes $\sqrt{2}$ as large.
(E) It becomes half as large.

$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$

$f \propto \frac{1}{\sqrt{L}}$; $L_2 = 2L_1$; $\frac{f_2}{f_1} = \frac{1/\sqrt{L_2}}{1/\sqrt{L_1}} = \sqrt{\frac{L_1}{L_2}} = \sqrt{\frac{L_1}{2L_1}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

A8. A string is strung horizontally with a fixed tension. A wave of frequency 100 Hz is sent along the string, and it has a wave speed of 50.0 m/s. Then a second wave, of frequency 200 Hz, is sent along the string. What is the wave speed of the second wave?

B

(A) 25.0 m/s (B) 50.0 m/s (C) 70.7 m/s (D) 100 m/s (E) 125 m/s

$v = \sqrt{\frac{F}{\mu}} =$ constant

A9. As you travel down the highway in your car, an ambulance approaches you from the rear at a high speed sounding its siren at a frequency of 500 Hz. Which one of the following statements is **TRUE**?

X

NONE.

approaching source $\Rightarrow f_o > f_s$

driver hears 500 Hz

(A) You hear a frequency less than 500 Hz.
(B) You hear a frequency equal to 500 Hz.
(C) You and the ambulance driver both hear a frequency greater than 500 Hz.
(D) You hear a frequency greater than 500 Hz, whereas the ambulance driver hears a frequency lower than 500 Hz.
(E) You hear a frequency less than 500 Hz, whereas the ambulance driver hears a frequency of 500 Hz.

A10. A point source broadcasts sound into a uniform medium. An observer moves away from the source at a certain speed. If the power is increased by a factor of 3 and the distance from the source is also tripled, what is the resulting change in decibel level?

E

$I = \frac{P}{A}$; $I_1 = \frac{P_1}{A_1} = \frac{P_1}{4\pi r_1^2}$

$I_2 = \frac{P_2}{A_2} = \frac{3P_1}{4\pi(3r_1)^2} = \frac{P_1}{4\pi r_1^2} \cdot \frac{3}{9} = \frac{1}{3}I_1$

(A) The decibel level goes down by more than 10 dB.
(B) The decibel level goes up by less than 10 dB.
(C) There is no change in the decibel level.
(D) The answer cannot be determined because the speed of the observer is not known.
(E) The decibel level goes down by less than 10 dB.

less than a factor of 10 change, $\therefore$ less than a 10 dB change

A11. During a fine Saskatoon winter, the temperature goes from 0 °C one day to -35 °C the next day. You hear a loud noise coming from 1 km away. Which one of the following statements is **TRUE**?

B

$v = 331\,m/s\sqrt{\frac{T}{273K}}$

(A) The noise reaches you more quickly by 0.21 seconds during the cold day.
(B) The noise reaches you more slowly by 0.21 seconds during the cold day.
(C) The noise reaches you more quickly by 0.44 seconds during the cold day.
(D) The noise reaches you more slowly by 0.44 seconds during the cold day.
(E) There is no difference in the arrival times of the noise.

$v_0 = 331\,m/s$

$v_{-35} = 309\,m/s$

$t_0 = 1000m/331m/s = 3.02s$; $t_{-35} = 1000m/309m/s = 3.24s$

A12. Two speakers are producing identical in-phase sound waves of intensity I and wavelength λ. Choose the phrase that best completes the following sentence: "If you are a distance r from one speaker and a distance $r - \lambda$ from the other speaker, then…

A

(A) you are at a position of constructive interference and the intensity of the sound arriving from the speaker at a distance $r - \lambda$ is greater than the intensity of the sound from the other speaker."
(B) you are at a position of constructive interference and the intensity of the sound arriving from each speaker is the same."
(C) you are at a position of destructive interference and you hear no sound at your location."
(D) you are at a position of constructive interference and the intensity of the sound arriving from the speaker at a distance r is greater than the intensity of the sound from the other speaker."
(E) you are at a position of destructive interference and you hear sound of low intensity."

A13. A standing wave is set up in a 200-cm string fixed at both ends. The string vibrates in 5 distinct segments when driven by a 120-Hz source. What is the wavelength of the standing wave?

D

(A) 10 cm (B) 20 cm (C) 40 cm (D) 80 cm (E) 100 cm

each segment is $\frac{1}{2}\lambda$. $\therefore L = 5(\frac{1}{2}\lambda) = \lambda = \frac{2}{5}L = \frac{2}{5}(200cm) = 80cm$

continued on page 4...

A14. A hollow pipe (such as an organ pipe open at both ends) is made to go into resonance at frequency f_{open}. One end of the pipe is now covered and the pipe is again made to go into resonance, this time at frequency f_{closed}. Both resonances are first harmonics (fundamentals). How do these two resonances compare?

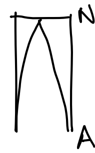
B

$f_c = \frac{v}{4L} = \frac{1}{2} f_o$

(A) They are the same.
(B) $f_{closed} = ½ f_{open}$
(C) $f_{closed} = 2 f_{open}$
(D) $f_{closed} = \sqrt{2}\ f_{open}$
(E) $f_{closed} = (3/2) f_{open}$

$L = \frac{1}{2}\lambda_o$ $f_o = \frac{v}{2L}$

$\lambda_o = 2L$



$L = \frac{1}{4}\lambda_c$

$\lambda_c = 4L$

A15. I stretch a rubber band and "plunk" it to make it vibrate at its fundamental frequency. I then stretch it to twice its length and make it vibrate at the fundamental frequency once again. The rubber band is made so that doubling its length doubles the tension and reduces the mass per unit length to half of its original value. The new frequency will be related to the old by a factor of:

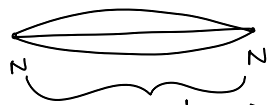
A

(A) 1.0 (B) 1.4 (C) 2.0 (D) 2.5 (E) 4.0

$L_B = 2L_A$

$F_B = 2F_A$

$\mu_B = \frac{1}{2}\mu_A$



$L = \frac{1}{2}\lambda_1 \Rightarrow \lambda_1 = 2L \Rightarrow f_1 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$

$f_{1B} = \frac{1}{2L_B}\sqrt{\frac{F_B}{\mu_B}} = \frac{1}{2(2L_A)}\sqrt{\frac{2F_A}{\frac{1}{2}\mu_A}} = \frac{1}{2(2L_A)}\sqrt{\frac{4F_A}{\mu_A}} = \underbrace{\frac{\sqrt{4}}{2}}_{=1} \cdot \underbrace{\frac{1}{2L_A}\sqrt{\frac{F_A}{\mu_A}}}_{f_{1A}}$

$f_{1B} = f_{1A}$

PART B

ANSWER <u>THREE</u> OF THE PART B QUESTIONS ON THE FOLLOWING PAGES AND INDICATE YOUR CHOICES ON THE COVER PAGE.

FOR EACH OF YOUR CHOSEN PART B QUESTIONS ON THE FOLLOWING PAGES, GIVE THE COMPLETE SOLUTION AND ENTER THE FINAL ANSWER IN THE BOX PROVIDED.

THE ANSWERS MUST CONTAIN THREE SIGNIFICANT FIGURES AND THE UNITS MUST BE GIVEN.

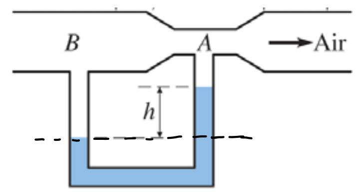
<u>SHOW AND EXPLAIN YOUR WORK</u> – NO CREDIT WILL BE GIVEN FOR ANSWERS ONLY.

EQUATIONS NOT PROVIDED ON THE FORMULAE SHEET MUST BE DERIVED.

USE THE BACK OF THE PREVIOUS PAGE FOR YOUR ROUGH WORK.

continued on page 5...

B1. For the purposes of this question, you may assume that air is an ideal, incompressible fluid. Air is flowing into a Venturi meter (see diagram). The narrow section of the pipe at point *A* has a radius that is ½ of the radius of the larger section of the pipe at point *B*. You may assume that the density of the air is constant. The U-shaped tube is filled with water.



(a) If the air speed at *B* is υ, how fast is the air moving at point *A*, in terms of υ? (3 marks)

Continuity of Flow, assumed constant density.

$\boxed{4\upsilon}$

$$\therefore A_A \upsilon_A = A_B \upsilon_B \Rightarrow \pi r_A^2 \upsilon_A = \pi r_B^2 \upsilon \; ; \; r_A = \tfrac{1}{2} r_B$$

$$\pi \left(\tfrac{1}{2} r_B\right)^2 \upsilon_A = \pi r_B^2 \upsilon$$

$$\tfrac{1}{4} \upsilon_A = \upsilon \Rightarrow \upsilon_A = 4\upsilon$$

(b) The density of air is 1.29 kg/m^3 and the density of water is 1.00×10^3 kg/m^3. The difference in the height, *h*, of the water in the arms of the manometer (the U-shaped tube) is 1.75 cm. Calculate the numerical value of υ. If you did not obtain an answer for (a), use a value of 2υ. (4 marks)

Bernoulli's Principle applies

$\boxed{4.21 \text{ m/s}}$

$$P_A + \tfrac{1}{2}\rho \upsilon_A^2 + \rho g y_A = P_B + \tfrac{1}{2}\rho \upsilon_B^2 + \rho g y_B$$

$$P_B = P_A + \rho_w g h \; ; \; y_A = y_B$$

$$\therefore \cancel{P_A} + \tfrac{1}{2}\rho (4\upsilon)^2 = \cancel{P_A} + \rho_w g h + \tfrac{1}{2}\rho \upsilon^2$$

$$16 \rho \upsilon^2 = 2\rho_w g h + \rho \upsilon^2$$

$$15 \rho \upsilon^2 = 2\rho_w g h$$

$$\upsilon = \sqrt{\frac{2\rho_w g h}{15\rho}} = \sqrt{\frac{2(1.00\times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.0175 \text{ m})}{15(1.29 \text{ kg/m}^3)}}$$

$$\upsilon = 4.21 \text{ m/s}$$

(c) The radius of the larger section of pipe is 5.00 cm. Calculate the volume flow rate (in m^3/s) of air through the pipe. If you did not obtain an answer for (b), use a value of 4.50 m/s. (3 marks)

$$Q = A\upsilon$$

$\boxed{3.31 \times 10^{-2} \text{ m}^3/\text{s}}$

$$Q = \pi r^2 \upsilon$$

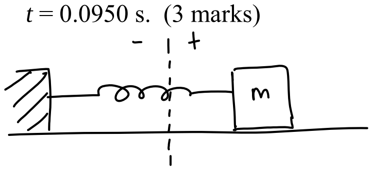
$$Q = \pi (0.0500 \text{ m})^2 (4.21 \text{ m/s}) = 3.31 \times 10^{-2} \text{ m}^3/\text{s}$$

continued on page 6...

B2. Consider an object with a mass of 375 g sitting on a horizontal, frictionless surface. The object is attached to an ideal, horizontal spring that is attached to the wall at its other end. The object is pulled away from its equilibrium position, in the positive direction, and released. The resulting motion of the object is described by the following equation:

$$x = (5.00 \text{ cm}) \cos (35.3\ t) \text{ where } t \text{ is in seconds}$$

(a) Calculate the position (distance and sign) of the object relative to its equilibrium position at $t = 0.0950$ s. (3 marks)



$x = A \cos(2\pi f t)$

$\boxed{-4.89 \text{ cm}}$

$\therefore A = 5.00 \text{cm}$

$2\pi f = \omega = 35.3 \text{ rad/s}$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{35.3 \text{ rad/s}} = 0.178 \text{ s}$

$x = 5.00\text{cm} \cos\left((35.3 \text{ rad/s})(0.0950\text{s})\right) = -4.89 \text{ cm}$

(b) Calculate the magnitude of the force of the spring on the object at $t = 0.095$ s. If you did not obtain an answer for (a), use a value of 0.0475 m for the magnitude of the displacement. (4 marks)

$|F| = k|x|$ and $\omega = 2\pi f = \sqrt{\frac{k}{m}}$

$\boxed{22.9 \text{ N}}$

$k = \omega^2 m = (35.3 \text{ rad/s})^2 \cdot (0.375 \text{ kg}) = 467 \text{ kg/s}^2$

$|F| = (467 \text{ kg/s}^2)(0.0489 \text{m}) = 22.9 \text{N}$

(c) Calculate the velocity (speed and sign) of the object at $t = 0.0950$ s. (3 marks)

$\boxed{+37.1 \text{ cm/s}}$

$v = -A\omega \sin(2\pi f t)$

$v = -(5.00\text{cm})(35.3 \frac{\text{rad}}{\text{s}}) \sin\left((35.3 \text{ rad/s})(0.0950\text{s})\right)$

$v = +37.1 \text{ cm/s}$

CHECK:

0.095 s is just a bit greater than $\frac{1}{2}T$, $\therefore$ the object has just passed max. negative displacement and is moving in the positive direction.

continued on page 7...

B3. A microphone is attached to a spring suspended from a ceiling (see diagram). Directly below on the floor is a stationary 441 Hz source. The microphone vibrates up and down in SHM with a period of 2.00 s. The difference between the maximum and minimum sound frequencies detected by the microphone is 2.10 Hz. The speed of sound is 343 m/s. *Hint: The maximum sound frequency is detected when the microphone is moving toward the source at maximum speed and the minimum sound frequency is detected when the microphone is moving away from the source at maximum speed. Also, you are given that* $f_{omax} - f_{omin} = 2.10$ Hz.

(a) Calculate the maximum speed of the microphone. (6 marks)

$$f_{o_{max}} = f_s\left(\frac{v + |v_m|}{v}\right); \quad v_m = \text{max. speed of microphone}$$

$\boxed{0.817 \text{ m/s}}$

$$f_{o_{min}} = f_s\left(\frac{v - |v_m|}{v}\right)$$

$$f_{o_{max}} - f_{o_{min}} = f_s\left(1 + \frac{|v_m|}{v}\right) - f_s\left(1 - \frac{|v_m|}{v}\right)$$

$$f_{o_{max}} - f_{o_{min}} = \cancel{f_s} + f_s\frac{|v_m|}{v} - \cancel{f_s} + f_s\frac{|v_m|}{v} = 2f_s\frac{|v_m|}{v}$$

$$\frac{v\left(f_{o_{max}} - f_{o_{min}}\right)}{2f_s} = |v_m|$$

$$|v_m| = \frac{(343 \text{ m/s})(2.10 \text{ Hz})}{2(441 \text{ Hz})} = 0.817 \text{ m/s}$$

(b) Calculate the amplitude of the simple harmonic motion of the microphone. If you did not obtain an answer for (a), use a value of 0.850 m/s. (4 marks)

$T = 2.00$ s

$\boxed{0.260 \text{ m}}$

$|v_m| = 0.817$ m/s

SHM: $v = -A\omega \sin(2\pi f t) \Rightarrow |v_m| = A\omega$ and $\omega = \frac{2\pi}{T}$

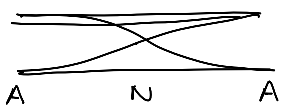
$$\therefore |v_m| = A\frac{2\pi}{T} \Rightarrow A = \frac{|v_m| T}{2\pi}$$

$$A = \frac{(0.817 \text{ m/s})(2.00 \text{ s})}{2\pi} = 0.260 \text{ m}$$

continued on page 8...

B4. An underground train station is connected to the outside through a tunnel. Some machinery is producing noise at a frequency dominated by 62.00 Hz and a second machine operating nearby has a strong noise peak at 58.00 Hz. The resulting beat is greatly amplified by the tunnel through resonance. That is, the beat frequency corresponds to one of the resonance modes of the tunnel. This makes a very annoying low frequency vibration for the passengers awaiting the next train.

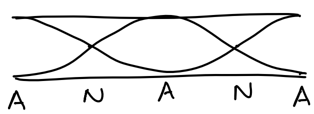
(a) Consider the tunnel to be an air pipe open at both ends and derive an expression for the possible natural (resonance) frequencies. (3 marks)



$$\underbrace{}_{L = \frac{1}{2}\lambda_1 \Rightarrow \lambda_1 = 2L} \qquad f_1 = \frac{v}{2L}$$

$$\boxed{f_n = n\left(\frac{v}{2L}\right)}$$

Next resonance mode:



$$\Rightarrow L = 2\left(\tfrac{1}{2}\lambda\right)$$

$$\lambda = L \Rightarrow f = \frac{v}{L} = 2\left(\frac{v}{2L}\right) = f_2 = 2f_1$$

In general,

$$f_n = n\left(\frac{v}{2L}\right)$$

(b) Calculate the frequency of the beats produced by the machinery. (2 marks)

$$f_b = |f_2 - f_1|$$

$$f_b = |62.00\text{ Hz} - 58.00\text{ Hz}|$$

$$f_b = 4.00\text{ Hz}$$

$$\boxed{4.00\text{ Hz}}$$

(c) You know that the tunnel is at least 100 m long, and knowing that the speed of sound was 344 m/s on that particular day, you figured that you could get a better estimate for the length of the tunnel, given the resonance condition. From the beat frequency and the knowledge that the tunnel is behaving like an air pipe open at both ends and is somewhat longer than 100 m, calculate which harmonic of the tunnel (air pipe) is being excited (i.e. which harmonic corresponds to the beat frequency) and calculate a more precise estimate for the length of the tunnel. If you did not obtain an answer for (b), use a value of 5 Hz. (5 marks)

harmonic:	3
length:	129 m

n is an integer

$v = 344$ m/s

$f_n = 4.00$ Hz

$L \geq 100$ m

$$f_n = n\left(\frac{v}{2L}\right)$$

$$n = \frac{f_n \cdot 2L}{v}$$

$$n \geq \frac{(4.00\text{ Hz})(2)(100\text{ m})}{344\text{ m/s}} = 2.3$$

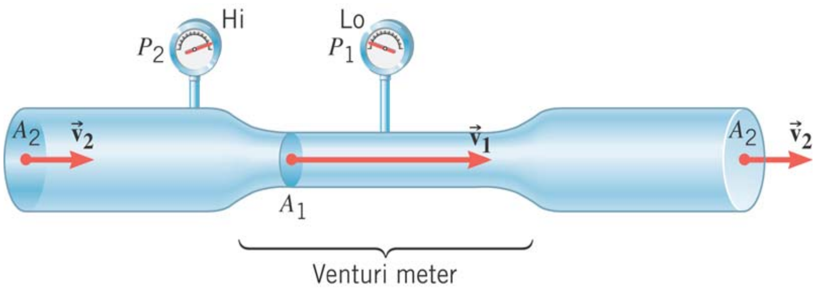
n must be an integer, so $n = 3$

Using $n = 3$ and solving for L:

$$L = \frac{nv}{2f_n} = \frac{3(344\text{ m/s})}{2(4.00\text{ Hz})} = 129\text{ m}$$

END OF EXAMINATION

B1. A Venturi meter is a device used for measuring the speed of a fluid within a pipe. A non-viscous fluid flows with a speed υ_2 through a horizontal section of pipe whose cross-sectional area is $A_2 = 0.0700\ \text{m}^2$. The fluid has a density of $1.30 \times 10^3\ \text{kg/m}^3$. The Venturi meter has a cross-sectional area of $A_1 = 0.0500\ \text{m}^2$, and has been substituted for a section of the larger, main pipe as shown in the figure below.



(a) The pressure difference, $P_2 - P_1$, between the two sections of pipe, is measured to be 1.20×10^2 Pa. Calculate υ_2, the flow speed in the main pipe. *Hint: Start by deriving an expression relating υ_1 to υ_2.* (7 marks)

From Continuity of Flow, $Q = A_1\upsilon_1 = A_2\upsilon_2$

$$\therefore \upsilon_1 = \frac{A_2\upsilon_2}{A_1} = \frac{0.0700\,\text{m}^2}{0.0500\,\text{m}^2}\cdot\upsilon_2 = 1.40\upsilon_2$$

From Bernoulli's Equation, $P_1 + \frac{1}{2}\rho\upsilon_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho\upsilon_2^2 + \rho g y_2$

horizontal pipe $\Rightarrow$ $y_1 = y_2$, so $P_1 + \frac{1}{2}\rho\upsilon_1^2 = P_2 + \frac{1}{2}\rho\upsilon_2^2$

Substituting for υ_1:

$$P_1 + \frac{1}{2}\rho(1.40\,\upsilon_2)^2 = P_2 + \frac{1}{2}\rho\upsilon_2^2$$

$$P_1 + \frac{1}{2}\rho\cdot 1.96\upsilon_2^2 = P_2 + \frac{1}{2}\rho\upsilon_2^2$$

$$\frac{1}{2}\rho(1.96\,\upsilon_2^2 - \upsilon_2^2) = P_2 - P_1$$

$$\frac{1}{2}\rho(0.96\,\upsilon_2^2) = P_2 - P_1$$

$$\upsilon_2 = \sqrt{\frac{2(P_2 - P_1)}{\rho(0.96)}} = \sqrt{\frac{2(1.20\times10^2\,\text{Pa})}{(1.30\times10^3\,\text{kg/m}^3)(0.96)}} = 0.439\ \text{m/s}$$

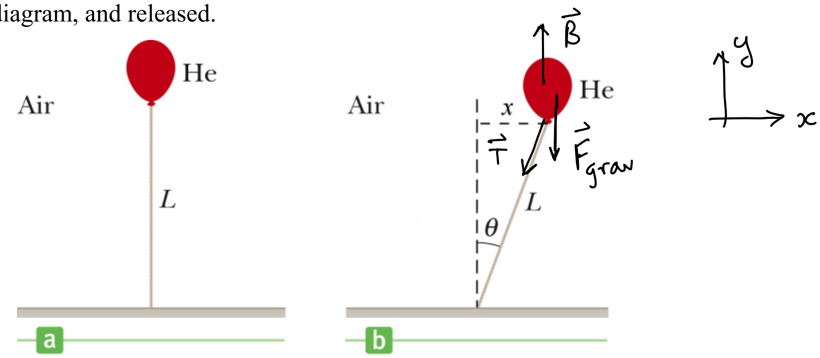
(b) Calculate the volume flow rate (in m^3/s) through the pipe. If you did not obtain an answer for (a), use a value of 0.500 m/s. (3 marks)

Q = volume flow rate $= A_1\upsilon_1 = A_2\upsilon_2$

$$Q = A_2\upsilon_2 = (0.0700\,\text{m}^2)(0.439\,\text{m/s}) = 3.07\times10^{-2}\ \text{m}^3/\text{s}$$

continued on page 6...

B2. A balloon filled with helium is tied to a light string of length L. The string is tied to the ground, forming an "inverted" pendulum. The balloon is displaced slightly from equilibrium, as shown in part (b) of the diagram, and released.



(a) Draw the three forces acting on the balloon in part (b) of the diagram. (3 marks)

(b) For small θ, it is a good approximation to consider the balloon to still be in equilibrium in the vertical direction (this is equivalent to assuming that $\cos\theta$ is very close to 1). Using this approximation, derive the expression for the tension T in terms of the mass of the balloon, m, the density of air, ρ_{air}, the volume of the balloon, V, and the acceleration due to gravity, g. (3 marks)

Equilibrium in the vertical direction:

$$\boxed{T = \rho_{air} g V - mg}$$

$$\sum F_y = 0$$

$$+B - F_{grav} - T\cos\theta = 0$$

$$\rho_{air} g V - mg - T\cos\theta = 0$$

$$\rho_{air} g V - mg = T\cos\theta$$

$$T = \frac{\rho_{air} g V - mg}{\cos\theta} \quad \text{and} \quad \cos\theta \simeq 1 \Rightarrow T = \rho_{air} g V - mg$$

(c) To a good approximation, the net force on the balloon will be the horizontal component of the tension. Show that this is a Hooke's Law force and derive the expression for the "spring constant" in terms of the density of air, ρ_{air}, the volume of the balloon, V, the mass of the balloon, m, the length of the string, L, and the acceleration due to gravity, g. (4 marks)

$$\boxed{k = \frac{\rho_{air} g V - mg}{L}}$$

$$\underset{\shortparallel}{\sum \vec{F}} \simeq T_x = -T\sin\theta$$

$$F_{net} = -(\rho_{air} g V - mg)\sin\theta$$

$$F_{net} = -(\rho_{air} g V - mg)\frac{x}{L}$$

$$F = -\underbrace{\left(\frac{\rho_{air} g V - mg}{L}\right)}_{k} x \quad \text{has the form of } F = -kx$$

$\therefore$ Hooke's Law force

continued on page 7...

B3. Two train whistles have identical frequencies of 180 Hz. When one train is at rest in the station and the other is moving nearby, a commuter standing on the station platform hears beats with a frequency of 5.00 Hz when the whistles operate together. The speed of sound on that particular day is 343 m/s.

(a) If the moving train is coming toward the observer, and given the 5.00 Hz beat, calculate the frequency that the observer detects for the moving train's whistle. (2 marks)

source moving toward observer $\Rightarrow f_o > f_s$

$$f_b = |f_2 - f_1|$$

$$\therefore f_o = f_s + f_b = 180\,\text{Hz} + 5\,\text{Hz} = 185\,\text{Hz}$$

$\boxed{185 \text{ Hz}}$

(b) Calculate the speed of the moving train. If you did not obtain an answer for (a), use a value of 182 Hz. (3 marks)

$$f_o = f_s\left(\frac{v}{v - v_s}\right);$$ v_s +ve for source moving toward observer

$$f_o v - f_o v_s = f_s v$$

$$f_o v - f_s v = f_o v_s$$

$$v_s = v\left(\frac{f_o - f_s}{f_o}\right) = 343\,\text{m/s}\left(\frac{185\,\text{Hz} - 180\,\text{Hz}}{185\,\text{Hz}}\right) = 9.27\,\text{m/s}$$

$\boxed{9.27 \text{ m/s}}$

(c) If the moving train is moving away from the observer, and given the 5.00 Hz beat, calculate the frequency detected by the observer for the moving train's whistle. (1 mark)

source moving away from observer, $f_o < f_s$

$$\therefore f_o = 180\,\text{Hz} - f_b = 180\,\text{Hz} - 5.00\,\text{Hz}$$

$$f_o = 175\,\text{Hz}$$

$\boxed{175 \text{ Hz}}$

(d) Calculate the speed of the moving train for the moving-away situation. If you did not obtain an answer for (c), use a value of 178 Hz. (3 marks)

source moving away $\Rightarrow v_s$ is −ve, $= -|v_s|$

$$f_o = f_s\left(\frac{v}{v - (-|v_s|)}\right) = f_s\left(\frac{v}{v + |v_s|}\right)$$

$$f_o v + f_o|v_s| = f_s v$$

$$f_o v - f_s v = -f_o|v_s|$$

$$|v_s| = \left(\frac{f_o - f_s}{-f_o}\right)v$$

$$|v_s| = \left(\frac{175\,\text{Hz} - 180\,\text{Hz}}{-175\,\text{Hz}}\right)343\,\text{m/s} = 9.80\,\text{m/s}$$

$\boxed{9.80 \text{ m/s}}$

(e) If the two trains were both moving toward the observer, but from opposite directions and at 16.7 m/s, what would be the beat frequency? (1 mark)

both trains moving toward observer at same speed would result in both observed frequencies being equal

$$\therefore f_{beat} = 0$$

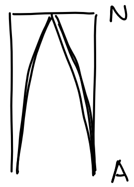
$\boxed{0\text{, no beats}}$

continued on page 8...

B4. An organ pipe of length 0.652 m, open at one end and closed at the other, has a fundamental resonance frequency of 137 Hz.

(a) Calculate the temperature of the air in the organ pipe. (4 marks)

FUNDAMENTAL



$\boxed{318\text{ K}}$

$L = \frac{1}{4}\lambda_1$ $\quad f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$

$\lambda_1 = 4L$ $\quad v = 4L \cdot f_1 = 4(0.652\text{m})(137\text{ Hz})$

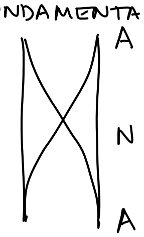
$v = 357\text{ m/s}$

$$v = 331\text{ m/s}\sqrt{\frac{T}{273\text{K}}} \Rightarrow \left(\frac{v}{331\text{ m/s}}\right)^2 \cdot 273\text{K} = T$$

$$T = \left(\frac{357\text{ m/s}}{331\text{ m/s}}\right)^2 \cdot 273\text{ K} = 318\text{ K}$$

(b) Calculate the length of a second organ pipe, open at both ends, that will also have a fundamental resonance frequency of 137 Hz when used where the air is the same temperature as that in the first pipe. (3 marks)

FUNDAMENTAL



$\boxed{1.30\text{ m}}$

$L_o = \frac{1}{2}\lambda_1$ $\quad f_1 = \frac{v}{2L_o}$ $\quad$ from (a), $f_1 = \frac{v}{4L_c}$

$\therefore 2L_o = 4L_c$ because f_1 and v are the same for both situations

$L_o = 2L_c$

$L_o = 2(0.652\text{m}) = 1.30\text{ m}$

Alternatively,

$$f_1 = \frac{v}{2L_o} \Rightarrow L_o = \frac{v}{2f_1} = \frac{357\text{ m/s (from (a))}}{2(137\text{ Hz})} = 1.30\text{ m}$$

(c) If the air in the second organ pipe cools to 0 °C (273 K), calculate the beat frequency between the sound of the second organ pipe and the 137 Hz sound produced by the first organ pipe. If you did not obtain an answer for (b), use a value of 1.30 m. (3 marks)

$f_{open} = \frac{v}{2L_o}$; v at $0°C = 331\text{ m/s}$

$\boxed{9.69\text{ Hz}}$

$$f_{beat} = 137\text{ Hz} - \frac{v}{2L_o} = 137\text{ Hz} - \frac{331\text{ m/s}}{2(1.30\text{m})} = 9.69\text{ Hz}$$

END OF EXAMINATION