Physics 115 Problem Solving Technique

Many students acquire bad problem solving habits in high school, which get them into trouble in university. It is important in this course to adopt good problem solving technique at the outset, since it will come in very handy later in the year. The technique to be used in Physics 115 is described below, with an example.

1. **Read the problem carefully**. This may sound obvious but it is easy to read the problem quickly, and misunderstand it – especially if you are under the pressure of writing a test. You should read the problem at least twice, visualise the situation, and identify what is being asked for.

2. **Make a sketch**. You should always try to do this, although there will be some situations in which a sketch is not relevant. On tests and examinations, marks will be deducted for diagrams which are missing or incorrect; sometimes you will get part marks for a diagram even if your solution is totally wrong. It doesn't have to be a work of art, just good enough to indicate what's happening.

   Many problems require that positions be mapped using a coordinate system. You should indicate your chosen coordinate system on the diagram. You can choose any coordinate system you like as long as you stick to it throughout the solution. Often the most convenient choice of coordinate system will be obvious, but sometimes you will have to choose one according to your personal taste. Whatever system you choose, indicate it clearly on your diagram: this will reduce errors and will also make the solution easier to mark.

3. **Identify the given information and the information being asked for**. Write a list of the known quantities: some of these are explicitly stated in the question, others are implied in the question (e.g., initially at rest), while still others may result from your choice of coordinate system (e.g., initial position). Be sure that the signs of these quantities are consistent with your coordinate system. You may wish to convert the units of some of the known quantities if they are inconsistent. Put symbols on the diagram corresponding to each of these quantities where appropriate. Finally, identify what quantity you need to determine and what symbol you will use.

4. **Identify the physical relationship between the variables**. This is frequently the hardest step. You should think about the question, examine the diagram, and identify which physics principle determines what will happen in the given situation. There is no "trick" to this other than understanding the course material. Then write down the appropriate equation or set of equations, and verify that you have an equal number of equations and unknowns. If you have fewer equations than unknowns, then you need another equation; you must find another relationship between the variables.

5. **Solve the equation(s) algebraically (symbolically) for the unknown quantity**. You should always solve the equation algebraically first: do not substitute numbers at this stage. This may be contrary to what you are used to from high school, but there are two reasons for doing this: first, it will reduce the number of errors you make; and second, sometimes quantities will cancel out. Solve the equation to get the unknown quantity alone on the left hand side, and a combination of known quantities on the right hand side.

6. **Substitute the known quantities and calculate the answer**. Substitute the known quantities into the right hand side. Make sure you substitute both the number and the units.

   Work out the units of the answer first. (Remember, units multiply and divide just like algebraic variables, so they can cancel each other when they appear on both the top and the bottom; also, any quantities which are added or subtracted must have the same units.) If the units you get are correct for the quantity you are trying to calculate, then you can have some confidence that your algebraic solution is correct; if not, then you have made an error somewhere. Go back and find it before proceeding. This is a very powerful method for detecting errors, but it only works if you know in advance the correct units for the quantity you are trying to calculate.

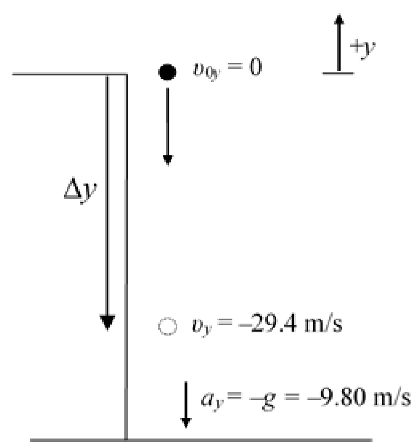
If the units are OK, then proceed with the numerical calculation to find your answer.

7. **Check your answer**. Having already checked the units in step 6, you should always check your answer in whatever ways you can think of. Is the sign correct? Is the magnitude roughly correct? This is not always easy to judge, but you should be able to recognise when an answer is wildly out. On one examination, for instance, we asked students to calculate the speed of protons passing through an electrical device. The correct answer was 1.38×10^6 m/s, but we had answers ranging from 7.22×10^{-7} m/s to 3.46×10^{15} m/s. It should be obvious that an answer less than 1 m/s or greater than the speed of light is very wrong, in which case you know to go back and look for your error.

Example: a one-dimensional, constant acceleration problem.

Question: A stone is dropped from the top of a tall building. Air resistance is negligible. Calculate the vertical displacement of the stone when its speed is 29.4 m/s.

Step 1. *Read the problem carefully and visualise the situation. The problem is talking about the motion of a stone from when it is dropped to when it reaches a certain speed. The stone has clearly moved between these two positions, which we will call the initial and final positions.*



Step 2. *Draw a diagram indicating the initial and final positions of the stone. The initial position is at the top of the building and the final position is the stone's location when its speed reaches 29.4 m/s. The final position must be somewhere below the top of the building but above the ground. Then choose a coordinate system: in this case we choose to put the origin at the top of the building (the initial position of the stone) and y positive upwards.*

Step 3. *One item of information is given explicitly in the problem: the speed of the stone at the final position; we call that* υ_y. *Two quantities are implied in the problem: first, the stone is "dropped", so its initial velocity* υ_{0y} *is zero; second, since air resistance is negligible, the acceleration is of magnitude g, directed downwards. List all three of these quantities. We also mark these on the diagram. List also the quantity being asked for: the displacement* Δy *(the difference between the final and initial positions).*

Given:
$$\upsilon_{0y} = 0$$
$$\upsilon_y = -\,29.4 \text{ m/s}$$
$$a_y = -\,g = -\,9.80 \text{ m/s}^2$$
$$\Delta y = ?$$
$$t = ?$$

Note that the velocity and the acceleration are both negative because both are in the negative direction, downwards. Note also that Δy *points downwards. (This means that we expect that our final answer for* Δy *should end up being negative, since we chose +y to be upward.) Note also that the time t is the time for* *the stone to go from the initial position to the final position. The problem tells us nothing about this, nor does the problem ask us to obtain it, but we list it for completeness.*

***Step 4**. The relevant physical principle is straightforward: the stone moves with constant acceleration since it falls straight downwards near the surface of the Earth with negligible air resistance. Look up the four constant acceleration equations in your notes, in your textbook, or on the formula sheet, and pick out the appropriate one. To find the displacement corresponding to the given initial and final velocities, find the equation containing Δx, a and υ_0 and υ, but not t. Replacing x with y for vertical motion, it is:*

$$\upsilon_y^2 = \upsilon_{0y}^2 + 2a_y\Delta y$$

We have one equation, with one unknown: Δy.

***Step 5**. Solve the equation for Δy. Note that $\upsilon_{0y}^2 = 0$. Drop this term, substitute $a_y = -g$, then divide both sides by $2g$:*

$$\upsilon_y^2 = 2(-g)\Delta y$$

$$\Rightarrow \frac{\upsilon_y^2}{2g} = -\Delta y$$

$$\Rightarrow \Delta y = -\frac{\upsilon_y^2}{2g}$$

Note that the unknown quantity, Δy, is alone on the left hand side of the equation, while the quantities on the right hand side are all known.

***Step 6**. Substitute the numbers and the units on the right hand side:*

$$\Delta y = -\frac{(-29.4 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)}$$

Check the units first. Since the velocity is squared, the units in the numerator are m^2/s^2. The units in the denominator are m/s^2.

$$\frac{\text{m}^2/\text{s}^2}{\text{m/s}^2} = \frac{\text{m}^2}{\text{s}^2} \times \frac{\text{s}^2}{\text{m}} = \text{m}$$

So the answer will be in m. Since we are calculating a displacement, these are the correct units. Now proceed to calculate the answer using a calculator:

$$\Delta y = -\,44.1 \text{ m}$$

***Step 7**. The units have already been checked in step 6. The sign is also correct, since the displacement must be negative; see the diagram. Finally, the answer does not seem to be either too large or too small: for the stone to acquire a speed of 29.4 m/s, the stone must travel more than 1 m but less than the height of a tall building. We conclude that the answer is reasonable.*