THE MEASUREMENT OF e/m BY THE BAINBRIDGE METHOD

OBJECT
The object of this experiment is to use the Bainbridge method to determine the electron charge-to-mass ratio.

DESCRIPTION OF APPARATUS
The apparatus consists of two main parts – an electron beam tube and a pair of Helmholtz coils which provide the variable magnetic field to control the circular path of the beam.
The filament of the electron tube emits a plentiful supply of electrons when heated. Surrounding the filament is a cylindrical anode which is maintained at 22.5 or 45.0 V positive with respect to ground (and with respect to the filament).

The electric field due to the applied voltage between filament and anode causes the electrons emitted from the filament to accelerate toward the cylindrical anode. A slit cut in the anode allows some of the accelerated electrons to emerge into the uniform magnetic field provided by the large Helmholtz coils. The radius of the electron’s circular trajectory can be adjusted by changing the magnetic field (this is accomplished by adjusting the current through the Helmholtz coils).
The electron beam emerges from the slit into a region of low pressure containing mercury vapour. When the electrons strike the mercury atoms, the mercury atoms are ionized or are excited (i.e. the outer electrons of the mercury atoms are activated into higher energy levels). When the excited mercury atoms return to the ground state, or when the ionized positive mercury ions recombine with free electrons, light is given off. This light is seen as a purple glow in the tube. (The process is very similar to the excitation of nitrogen and oxygen in the upper atmosphere by electrons spiralling down the earth’s magnetic field lines, producing the aurora.) The electrons which enter into collisions lose some energy, so the inside of the electron beam is fuzzy. However, the outer edge of the beam is sharp because it is formed by electrons at full energy that are just making their first collision. It is these outer electrons that are of interest in making the measurements.

**THEORY**

An electron emitted from the filament can be considered to have only a very small amount of kinetic energy. When the electrons accelerate toward and reach the anode at potential $V$ above the filament, the kinetic energy gained by the electrons (which equals the loss in electric potential energy) is given by

$$\frac{1}{2}mv^2 = eV$$  \hspace{1cm} (1)

where $m$ is the electron mass, $v$ is the electron speed at the anode, and $e$ is the electron charge. Solving for the speed of the electrons as they emerge from the slit in the anode,

$$v = \sqrt{\frac{2eV}{m}}$$  \hspace{1cm} (2)

When the electrons emerge from the slit into the magnetic field, they are subject to the magnetic force, of magnitude

$$F = evB\sin \theta$$  \hspace{1cm} (3)

which acts at right angles to the direction of motion (the velocity). When the electron velocity is perpendicular to the magnetic field (as it is in this experiment) the result is that the magnitude of the force is $evB$ and the electrons move in a circular path of radius $R$ with centripetal acceleration

$$a_c = \frac{v^2}{R}$$  \hspace{1cm} (4)

Applying Newton’s 2nd Law and using equations (3) and (4) and the condition that the electron velocity is perpendicular to the magnetic field yields

$$evB = \frac{m v^2}{R}$$  \hspace{1cm} (5)

Substituting (2) for $v$ and solving for $e/m$ yields

$$\frac{e}{m} = \frac{2V}{B^2 R^2}$$  \hspace{1cm} (6)
Equation (6) can be used to determine $e/m$ if, in addition to knowing the filament-anode potential difference, $V$, we can measure $R$ and $B$.

The radius $R$ is determined by adjusting the circular path of the beam until the sharp outer edge of the beam just reaches the far side of one of the crossbars, whose distances from the filament are accurately known. These filament-crossbar distances are the diameters of the circular paths.

The value of the magnetic field, $B$, can be calculated by measuring the current $I$ through the Helmholtz coils, the radius $a$ of the coils, the number of turns $N$ and then using the equation

$$B = \frac{8\mu_0 N (I - I_o)}{\sqrt{125} a}$$

(7)

where

$$\mu_0 = 4\pi \times 10^{-7} \text{T} \cdot \text{m/A}$$

and

$I_o =$ zero reference current

The zero reference current is a correction to account for the effects of the external magnetic field existing in the room. After aligning the equipment so that the Helmholtz coil magnetic field is antiparallel to the external magnetic field, the current through the Helmholtz coils that is required to cancel the external magnetic field is measured. This current is $I_o$, the zero reference current.

**Checkpoint 1 – ask the TA to review your pre-lab work**

**EXPERIMENT**

1. The magnetic field $B$ produced by the Helmholtz coils must be aligned (antiparallel) with the external magnetic field existing in the room. Alignment in the horizontal plane is accomplished by aligning the apparatus so that the tube points in the direction indicated by a compass needle placed on the base of the apparatus. Alignment in the vertical plane is accomplished by use of a dip compass. The dip compass shows the inclination from vertical of the external magnetic field. Note that the Helmholtz coils can be inclined and that the magnetic field produced by the Helmholtz coils is perpendicular to the plane of the coils. Place the dip compass on the base of the Helmholtz coil assembly. Carefully loosen the wing nut on the slotted metal guide attached to the coil assembly. While holding the dip compass on the coil assembly base, slowly incline the coil assembly until the dip compass reads 90°, indicating that the Helmholtz coil field will act along the same line as the external magnetic field. Ensure that the wing nut is tightened securely to hold the coil assembly in place at the proper incline.

2. There are two circuits to be wired – the electron tube circuit and the Helmholtz coils circuit. Connect these circuits as shown in the following circuit diagrams. Note that to obtain the proper Helmholtz coil magnetic field polarity the positive lead attaches to the bottom terminal of the bottom coil and the negative terminal of the voltage source attaches to the top lead of the top coil.
3. Adjust the anode circuit voltage divider rheostat (connected to Supply A) for minimum voltage. Set all other rheostats for maximum resistance.

Close the anode circuit knife switch. Slowly increase the anode circuit voltage divider rheostat until the voltmeter reads 22.5 V.

Ensure that the slit in the anode is facing magnetic east. Close the filament circuit knife switch and, by adjusting the filament circuit rheostat, slowly increase the filament current. As the filament current is increased, the filament will begin to glow incandescently. As the current is increased further, the purple electron beam will appear at about 3 A. Continue to increase the current slowly until the beam reaches the glass wall of the tube and deflects. **At no time should the current be allowed to exceed 4 A**, as this could cause the filament to burn out. If necessary, rotate the tube until the beam is horizontal.

4. When viewed from above, the electron beam will be seen to have a slight curvature toward the base of the tube. This is due to the force effect of the external magnetic field existing in the room.
The Helmholtz coil current required to compensate for this effect is the zero reference current $I_0$ and is determined as follows:

While viewing the beam from above, close the Helmholtz circuit knife switch and slowly increase the Helmholtz coil current. When the electron beam is completely straight (no curvature) there is no net force acting on it, and therefore the Helmholtz magnetic field is exactly cancelling the external magnetic field. Record the Helmholtz coil current value (the value of $I_0$).

5. Now increase the Helmholtz coil current so that the beam is deflected into a horizontal circular trajectory. For each of the five crossbars (starting at the inner bar) measure the Helmholtz coil current required for the sharp outer edge of the electron beam to be striking the outer edge of the crossbar.

6. Adjust the anode voltage to 45.0 V and repeat the procedure described above (including re-determining the zero reference current).

(7. Adjust the anode voltage to 35.0 V and repeat the procedure described above (including re-determining the zero reference current).)

**ANALYSIS**

Record the data and results in the table provided in the Worksheets package.

Calculate the values of magnetic field, $B$, and the corresponding experimental errors.

**Checkpoint 2 – ask the TA to review your magnetic field (and exp. error) calculation**

Calculate the 10 (15) values of $e/m$ and the corresponding experimental errors.

**Checkpoint 3 – ask the TA to review your $e/m$ (and exp. error) calculation**

For each set of five values of $e/m$ corresponding to a particular value of anode voltage, calculate the average $e/m$ value. Calculate the experimental error in this average value by calculating the average of the errors in the $e/m$ values.

Now calculate the average of these average values of $e/m$ and compare with the accepted value of $1.759 \times 10^{11}$ C/kg. (Calculate the experimental error in the overall average value by calculating the average of the errors in the $e/m$ values.)

**Checkpoint 4 – ask the TA to review your average $e/m$ value and its comparison with the accepted value**

**CONCLUSION**

Discuss the qualitative aspects of the experiment in addition to the numerical results. i.e. Did the magnetic field have the expected effect on the electron beam trajectory, how did changing the anode voltage affect the experiment, ...

**Checkpoint 5 – ask the TA to join your discussion of your Conclusion.**

**SOURCES OF ERROR**

As usual, think of as many factors as possible that likely affected the results of your experiment, but were not assigned an experimental error.

**Checkpoint 6 – ask the TA to join your discussion of your Sources of Error.**
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**DATA & RESULTS**

Number of turns in Helmholtz Coils, \( N = 72 \) turns

Radius of Helmholtz Coils, \( a = 0.330 \text{ m} \pm 0.001 \text{ m} \)

<table>
<thead>
<tr>
<th>Filament-Cross bar Distance (m) ± 1%</th>
<th>Beam Path Radius, ( R ) (m) ± 1%</th>
<th>Coil Current, ( I ) (±____A)</th>
<th>Net Current ( I - I_o ) (±____A)</th>
<th>Magnetic Field, ( B ) (× ( 10^{-4} ) T)</th>
<th>Charge-to-Mass Ratio, ( e/m ) (× ( 10^{11} ) C/kg)</th>
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</thead>
<tbody>
<tr>
<td>Anode Voltage, ( V ) (V):</td>
<td>0.0650 ±</td>
<td>0.0325</td>
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<tr>
<td>±</td>
<td>0.0780 ±</td>
<td>0.0390</td>
<td>±</td>
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<tr>
<td>Zero Reference Current, ( I_o ) (A):</td>
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<td>0.0450</td>
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<tr>
<td>Average ( e/m ):</td>
<td>0.1030 ±</td>
<td>0.0515</td>
<td>±</td>
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<tr>
<td>±</td>
<td>0.1150 ±</td>
<td>0.0575</td>
<td>±</td>
<td>±</td>
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