MATH 450/872 2024-2025

Algebraic Geometry through Moduli T1

Instructor:

Dr. Steven Rayan
rayan@math.usask.ca

Course Details

Math 450 CRN 91025
Math 872 CRN 91028

Schedule:

Term 1,
Monday's 5:30 PM-8:00 PM

Course Objective:

A major moment in mathematics and science was when we learned to describe geometries in terms of algebraic equations. For example, we now think about the unit circle interchangeably as its picture (a circle of radius 1 centred at a choice of origin in the plane) and as the algebraic equation x^2 + y^2 = 1. Without the ability to describe spaces algebraically as equations or relations in coordinates, much of the engineering or physics that we rely upon today would be impossible.

Mathematicians have, for a few centuries, been asking the inverse question as well: given an algebraic equation, or systems of such equations, what geometry do they describe? When the system of equations is entirely linear (i.e. each polynomial has degree 1), we recover what is now known as linear algebra, which gives a complete answer to this question: the geometries described by such systems are lines, planes, and hyperplanes. What happens when we look at higher degree systems? This question is almost never addressed in undergraduate mathematics, but is nonetheless very important for applications of both a theoretical and practical nature. Surprisingly, even when we keep our attention focused on linear systems, there are issues that come up that are not usually seen in linear algebra, such as when we try to classify all of the linear systems of a fixed type. Such classification problems are called "moduli problems" and they are an important source of new geometries.

We will see examples and applications of moduli spaces, and we will attempt to make contact between these ideas and real-world applications in computer vision, machine learning, data security, and so on.

Tentative Topics:

- What is Algebraic Geometry?
- Algebraic Geometry and Linear Algebra
- Group Actions and Moduli
- The Projective Line as a Motivating Example
- From Planes to Varieties
- Quivers and Quiver Varieties
- Applications of Algebraic Geometry to the Real World

Students Who May Be Interested:

- Mathematics,
- Statistics,
- Mathematical Physics,
- Physics,
- Computer Science

Other Information:

This course will likely appeal to anyone who did well in any of group theory, ring theory, Galois theory, introduction to topology, etc., but it is absolutely not necessary to have taken these. The only actual prerequisite for this course will be MATH 266 (Linear Algebra II). No higher knowledge in mathematics will be required.

Essentially, if you like linear algebra or geometry (or both), then you will likely enjoy this course. Students will be assessed through assignments (three, approximately one per month) and a final quiz. Students interested in pursuing a project as part of the course are welcome to discuss that with the instructor for recommendations in that regard.