## William Lowell Putnam Mathematical Competition 2012

The William Lowell Putnam Mathematical Competition is open to regularly enrolled undergraduates in colleges and universities in Canada and the United States who have not yet received a college degree.

This competition will be written on

## Saturday, 1 December, 2012 in two three-hour sessions: 09:00-NOON, 14:00-17:00

In 2011 a total of 4440 students from 572 colleges and universities in Canada and the United States participated in the Competition. There were teams from 460 institutions.

A high individual score in the competition is likely to result in the contestant receiving attractive offers of financial support to study towards graduate degrees in Mathematics, whereas a top score will win a Scholarship to the Mathematics graduate school of your choice.

The competition, which began in 1938, boasts a list of top-scorers that reads like a "who's who" of eminent Mathematicians, Statisticians as well as Scientists in other disciplines. Several Fields Medalists and Nobel Prize Winners have been top-scorers on the Putnam examination.

The U of S will compete in this event and in order to get ready for it interested problem-solvers are invited to attend weekly practice problem-solving sessions. Copies of previous papers will be made available at these sessions where we will (attempt to) solve problems together as a group. During the week between sessions it is expected that would-be contestants will attempt several problems and write up neat solutions to bring to the next session for discussion.

The first session will take place once we know who wishes to participate. An announcement will be made at that time.

## To express your interest please drop a message to Professor Brooke at 966-6089 or by e-mail at <u>brooke@math.usask.ca</u>

But please act without delay - no later than Wednesday, 3 October 2012 - to express your interest.

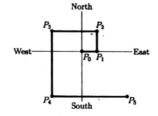
A copy of the 2011 Competition is posted on the wall outside Prof Brooke's office at 233 McLean Hall – please have a look to see if it is of interest.



## Problem A1

Define a growing spiral in the plane to be a sequence of points with integer coordinates  $P_0 = (0, 0), P_1, \dots, P_n$  such that  $n \ge 2$  and:

- The directed line segments P<sub>0</sub>P<sub>1</sub>, P<sub>1</sub>P<sub>2</sub>,..., P<sub>s-1</sub>P<sub>s</sub> are in the successive coordinate directions east (for P<sub>0</sub>P<sub>1</sub>), north, west, south, east, etc.
- The lengths of these line segments are positive and strictly increasing.



How many of the points (x, y) with integer coordinates  $0 \le x \le 2011$ ,  $0 \le y \le 2011$ cannot be the last point,  $P_x$  of any growing spiral?