I would like to thank Professor Joseph S. DeSalvo at the University of South Florida for introducing me to this research area and numerous exchanges on the topic.
THEORETICAL ANALYSIS ON VALUATION OF TIME SAVING

by

Mobinul Huq

ABSTRACT

The value of time (\textit{VOT}) is defined as the maximum amount of money people are willing to pay for an additional unit of time, while the value of time saving (\textit{VTS}) is defined as the maximum amount of money people are willing to pay for a reallocation of time between two alternative activities. This paper shows that the sign and amount of the \textit{VTS} depends on the magnitude of the differential effects of time-use on an individual's money constraint. While the existing models of value of travel time saving (\textit{VTTS}) hinge on the presence of a binding minimum travel requirement, this paper presents a choice-theoretic model where a positive \textit{VTTS} is derived without imposing a binding minimum travel requirement. This research suggests alternative ways of estimating \textit{VTTS} by examining market behaviour where people are observed to trade travel time for money. One way is estimation of the decrease in housing expenditures with travel time; another is estimation of the money-time trade-off in travelling between two fixed points in similar travel circumstances.

\textbf{Key Words}: Value of time (\textit{VOT}), value of time saving (\textit{VTS}), value of travel time saving (\textit{VTTS}).

\textbf{JEL Classifications}: D1, J2, R2, R4.
1. INTRODUCTION

In the goods-market context, the economic value of a commodity is defined as the maximum amount of money an individual is willing to pay for an additional unit of that commodity. Applying this definition to the time-allocation problem, the value of time (VOT) is defined as the maximum amount of money an individual is willing to pay for an additional unit of time. Given the fact that total amount of time available is fixed, which implies that any time saved in an activity must be allocated to some other activity(ies), the value of time saving (VTS) is defined as the maximum willingness to pay for reallocation of time between two alternative activities.

The traditional model of labour-leisure choice, formulated to explain hours of labour supplied to work, defines leisure as all activities other than market work. In addition, it is assumed that the marginal utility of market work time is identically zero, which means market work does not involve any satisfaction or dissatisfaction. One implication of these assumptions is that, when evaluated at the optimum point, there is a unique value of time which is equal to the market wage rate. In addition, the value of work time saving is equal to the wage rate which is also equal to the monetary value of the marginal utility of leisure.

In economic models dealing with the allocation of time among more than two alternatives, specifically in the valuation of travel time saving, the term value of time has been used in different contexts to mean different ideas. Recognizing the lack of well-defined concepts of values of time in the context of transportation, Small (1992, p. 36) wrote, “Among the most important quantities inferred from travel demand studies are the money values that people place on saving various forms of travel time. Loosely known as the value of time, this concept encompasses many specific measures....”

The first purpose of this paper is to review the theoretical literature on value of time in general and in the context of travel-time saving. Distinctions are made between three different concepts commonly used in relation to the valuation of time use; the utility (or process) effect, the consumption (or monetary) effect and the net (or total) benefit. It is argued that in the literature, VOT concept relates to the net benefits from time use and VTS refers to difference in the utility (process) effects of two alternative activities. To explore these relationships, a generalized time and goods allocation model is presented in the third section. At the optimized point while the monetary value of total benefits from time use to all activities are equalized at the VOT, the monetary value of utility effects are not equalized. This later result generates non zero benefits from time reallocation between alternative activities.
In the travel time context, through a review of the literature, it is shown that the value of travel-time saving (VTTS) critically hinges on the assumption of a ‘binding minimum travel time’ constraint that requires individuals to travel more than they would choose to travel without the constraint. It is not obvious, however, whether travel time is a constraint or a choice (Handy et al., 2005; Ory et al., 2004). In most situations, it is possible to reduce travel time by spending more money on travel goods, such as taking a taxi instead of walking, although that might not be the optimal choice. In the absence of a binding minimum travel-time requirement, VTTS is zero in the existing models. The latter part of this paper presents a model of travel time in a choice-theoretic framework by combining the standard urban housing location model with mode choice treated as a continuous variable. One implication of the proposed model is that there is a positive VTTS even when there is no binding minimum travel requirement. The main argument is that, although leisure is preferred to travel, people travel to obtain better terms of trade. This model treats commuting time as a choice variable in the same way as the standard literature treats work time as a choice variable.

The paper is organized as follows. The next section reviews the VOT in terms of the labour-leisure model, and the following sections presents a generalized model. The fourth section presents a discussion of the theoretical literature on VTTS. The fifth section presents the proposed residential choice model with mode choice and discusses its implications for travel-time valuation research. The paper ends with a concluding section.

2. VALUATION OF TIME

The Traditional Time Allocation Model

The traditional labour-supply model in its simplest form examines the problem of best allocation of total available time, $M$, between market work, $W$, and leisure, $L$, assuming that an individual is free to choose to work for any number of hours at a fixed wage rate, $w$. The quasi-concave utility function is defined as $u = u(X,L)$, where $X$ stands for market goods measured in terms of dollars. One notable feature of the model is that working time is not included in the utility function. An individual faces a time constraint, $M = L + W$, as well as a budget constraint which states that the total expenditure on market goods $X$ is equal to the sum of labour and non-labour income, $y$, $X = wW + y$.

This constrained optimization problem in Lagrangean form can be written as

$$\Lambda = u(X,L) + \lambda (wW + y - X) + \mu (M - W - T)$$

By the envelope theorem, the Lagrange multiplier $\mu = \partial \Lambda / \partial M$ represents the marginal utility of time $M$, while $\lambda = \partial \Lambda / \partial y$ represents the marginal utility of money. The value of time (VOT) can be defined as $(\partial \Lambda / \partial M) / (\partial \Lambda / \partial y) = (\mu / \lambda)$. From the first-order conditions for this constrained optimization problem (treating $W$ as a choice variable), one can derive the value of time (VOT) as

$$VOT = \frac{\mu}{\lambda} = \frac{u_L}{u_X} = \frac{\mu}{\lambda} = w$$
The issue of exclusion of work time from the utility function has been questioned by a number of researchers. Among others, Johnson (1966), Oort (1969), and Evans (1972) argued that there is no reason to assume that the marginal utility of work time is zero. Therefore, by including $W$ as an additional variable in the utility function, the utility function would be defined as $u(X, L, W)$. The value of time in this formulation can be derived as

$$VOT = \frac{u_L}{u_X} = \frac{w}{u_X} + \frac{w}{u_X}$$

Thus the value of time is no longer equal to the wage rate, but it would exceed (fall short of) the wage rate when positive (negative) utility is derived from work time.

Alternatively, the $u(X, L)$ function can be interpreted as the one where the time constraint has been substituted for $W$, that is $v(X, L, W) = v(X, L, M - L) = u(X, L)$. As correctly pointed out by Flemming (1973), this will imply,

$$\frac{u_L}{u_X} = \frac{v_L}{v_X} - \frac{v_w}{v_X} = MRS_{lx} - MRS_{wx}$$

When more than one time use activities are included in the utility function, such as $L$ and $W$ in the preceding, it is important that one keeps the following time-related concepts distinct.

*The monetary value of the marginal utility of time spent in activity $i$, $\beta_i$.\*

For an utility function with $n$ alternative activities, $u(X, T_1, \ldots, T_n)$, this concept can be defined as

$$\beta_i = MRS_{T_i X} = \frac{u_L}{u_X} \quad i = 1, \ldots, n.$$  

In this paper, this $MRS$ is referred to as the *utility (process) effect* of time-use and is denoted by $\beta_i$ for activity $i$. DeSerpa (1971) defined this concept as the *value of time as a commodity (VTC).*

In a model with work time incorporated into the utility function,

$$\beta_L = MRS_{LX} = \frac{u_L}{u_X} \quad \text{and} \quad \beta_w = MRS_{WX} = \frac{u_w}{u_X}$$

These utility effects are activity-specific and in general likely to vary across alternative uses of time at the optimal point.

*The consumption (monetary) effect of time spent in activity $i$, $\gamma_i$.\*

In addition to the direct effect on the utility level, time use also has a potential effect on utility through its effect on the budget constraint. The obvious case is the market work time which results in an increase in money income by an amount equal to the wage rate, resulting in an expansion in consumption possibilities, hence increased satisfaction. Among others, Jara-
Diaz (2003) examined this effect in the travel-time context, and Jiang and Morikawa (2005) referred to it as the value of consumption saving (VCS). In this paper, this effect is referred to as the consumption (monetary) effect of time-use in an activity and is denoted by $\gamma_i$. This effect is likely to vary across alternative time-use activities. Theoretically, however, the sign of this effect is indeterminate, which implies that it might not be an inflow but an outflow of money.

In other words, the consumption effect of the $i$th activity refers to the effect of activity $i$ on the availability of the numéraire good ($X$). This effect can be derived by differentiation the budget constraint in the form $X = X(T_1, ..., T_n)$. In the standard labour leisure choice model, $X = wW + y$ gives,

$$
\gamma_L = \frac{\partial X}{\partial L} = 0 \quad \text{and} \quad \gamma_y = \frac{\partial X}{\partial W} = w
$$

These effects are independent of the preference of an individual and can be derived from market data.

The net (total) benefit from time spent in activity $i$, $\alpha_i$.

$\alpha_i$ is defined as the net (or total) benefit generated from a change in time allocated to an activity $i$. In other words, it is the rate of change in utility with respect to $T_i$ measured in terms of money. The net benefit of time use is the sum of the utility effect and the consumption effect, namely $\alpha_i = \beta_i + \gamma_i$.

In the context of the labour-leisure choice model, these effects are

$$
\alpha_L = \frac{u_L}{u_X} \quad \text{and} \quad \alpha_w = \frac{u_w}{u_X} + w
$$

One property of the optimal point is that the net benefits from all alternative time-use activities are equalized ($= \alpha$), and there is no room for further utility gain through time reallocation. Any addition to total available time can be assigned to any of the available activities, and the resulting utility gain will be the same and equal to $\alpha$. Hence $\alpha_i$ can be interpreted as the value of time (VOT), which by the first-order conditions is equal to the ratio of the two Lagrange multipliers, $(\mu / \lambda)$. DeSerpa (1971) defined this as the value of time as a resource (VTR).

Willingness to pay to reallocate time from activity $i$ to $j$, $\rho_{i\rightarrow j}$.

Mathematically this value can be derived in the following way. Let the utility function be $u = u(X, T_1, ..., T_n)$ and the time constraint be $M - \sum T_i = 0$ where $T_i$ denotes time allocated to activity $i = 1, ..., n$.

Totally differentiating the utility function, assuming $M$ constant, produces

$$
du = \frac{du}{dX} dX + \frac{du}{dT_1} dT_1 + ... + \frac{du}{dT_n} dT_n
$$
Setting \( du = dT_k = 0 \) for all \( k = 1, \ldots, i-1, j+1, \ldots, n \), i.e. \( k \neq i \) or \( j \), and \( dT_j = -dT_i \) since \( M \) is a constant, gives

\[
0 = u_X \frac{dX}{dT_i} = u_i \frac{dT_i}{dT_i} + u_j (-dT_i) = u_j - u_i = MRS_{jX} = \beta_j - \beta_i = \rho_{j \rightarrow i}.
\]

In the commuting time context, DeSerpa (1971) defined such a difference between the marginal utility of leisure and of travel time, \( T \), as the value of travel-time saving (VTTS)

\[
VTTS = \frac{u_j - u_i}{u_X} = \rho_{j \rightarrow i}.
\]

Note that the marginal rate of substitution between two activities, \( MRS_{ji} = u_j / u_i \), a different concept, represents the rate at which an individual is willing to exchange time between two alternative activities, \( i \) and \( j \). If and only if \( u_j = u_i \), such a time reallocation will satisfy the time constraint. Otherwise utility constant time trade-off between two activities will violate the time constraint.

3. A GENERALIZED MODEL

An individual’s utility is assumed to depend on consumption of \((g+1)\) market goods (the numeraire good with price \( =1 \) is denoted by \( X_0 \)) and \((n+1)\) alternative time use activities (a base or reference activity is denoted by \( T_0 \)).

\[
u = u(X_0, X_1, \ldots, X_g, T_0, T_1, \ldots, T_n)
\]

The monetary value of the marginal utility of time use in activity \( i \) (\( \beta_i \)), is defined as,

\[
\beta_i = MRS_{iT} = \frac{u_T}{u_T} = \rho_{T \rightarrow i}, \quad i = 0, 1, \ldots, n
\]

The time constraint is given as,

\[
M - \sum_{i=0}^{n} T_i = 0.
\]

Following Evans (1972), it is assumed that any activity may be associated with monetary remuneration (inflow) or monetary commitment (outflow), such as working for pay or paying to go to a movie respectively. This amount is denoted by \( R_i \). A positive value implies that an individual receives \( R_i \) amount per unit of time spent in activity \( i \) while a negative value implies \( R_i \) expenditure per unit of time.
Following urban economics literature, goods/housing price is assumed to be affected by time use and market goods (for example, more commuting time and commuting goods lead to lower house price). In the generalized case any price may be affected by time use and goods, such as money back rebate by mail needs time and money/postage. That is $P_j (X_1, ..., X_g, T_0, ..., T_n)$. The money constraint can be written as:

$$y + \sum_{j=0}^{n} R_i T_i - X_0 - \sum_{j=1}^{g} P_j (X_1, ..., X_g, T_0, ..., T_n) X_j = 0$$

(3)

$y$ stands for non-wage income.

The consumption (monetary) effect of time use in activity $i$ ($y_i$) can be obtained by differentiating the following budget constraint,

$$X_0 = [y + \sum_{i=0}^{n} R_i T_i] - \sum_{j=1}^{g} P_j (T_0, ..., T_n) X_j$$

$$\gamma_i = \frac{\partial X_0}{\partial T_i} = R_i - \sum_{j=1}^{g} \frac{\partial P_j}{\partial T_i} X_j, \forall i = 0, 1, 2, ..., n$$

The net (total) benefit of time use in activity $i$ ($\alpha_i$), are

$$\alpha_i = \beta_i + \gamma_i = \frac{U_r}{U_x} + R_i - \sum_{j=1}^{g} \frac{\partial P_j}{\partial T_i} X_j, \forall i = 0, 1, 2, ..., n$$

This model can be extended further where $R_i$ are affected by time use (for example, commuting further for higher paid job), or allowing for home production of some goods.

An individual’s optimization problem is to maximize (1) subject to (2) and (3).

$$\Lambda = u(X_0, X_1, ..., X_g, T_0, T_1, ..., T_n) + \lambda[y + \sum_{i=0}^{n} R_i T_i - X_0 - \sum_{j=1}^{g} P_j (X_1, ..., X_g, T_0, ..., T_n) X_j] + \mu(M - \sum_{i=0}^{n} T_i)$$

The first-order conditions for maximization are,

$$\frac{\partial \Lambda}{\partial X_0} = u_{X_0} - \lambda = 0$$

$$\frac{\partial \Lambda}{\partial X_j} = u_{X_j} - \lambda (P_j + \sum_{i=1}^{g} \frac{\partial P_j}{\partial X_j} X_j) = 0, \forall j = 1, 2, ..., g$$
\[
\frac{\partial \Lambda}{\partial R_i} = u + \lambda R_i - \lambda \sum_{j=1}^{s} \frac{\partial P_j}{\partial T_i} X_j - \mu = 0, \forall i = 0, 1, 2, \ldots, n
\]

\[
\frac{\partial \Lambda}{\partial \lambda} = \sum_{i=0}^{n} R_i T_i - X_0 - \sum_{j=1}^{s} P_j (X_1, X_2, \ldots, X_s, T_0, \ldots, T_n) X_j = 0
\]

\[
\frac{\partial \Lambda}{\partial \mu} = M - \sum_{i=0}^{n} T_i = 0
\]

Some Implications of the FOCs:

**The Monetary Value of the Marginal Utility of Time Use (β)**

Divide the third FOCs by the first one,

\[
\beta_i = \frac{u_i}{u_X} = \frac{(\mu / \lambda) - R_i + \sum_{j=1}^{s} \frac{\partial P_j}{\partial T_i} X_j}{(\mu / \lambda) - \gamma_i}, \forall i = 0, 1, 2, \ldots, n
\]

\[(n+2) \text{ unknowns}, (\beta_0, \beta_1, \ldots, \beta_n, \mu / \lambda), \text{ with } (n+1) \text{ equations. Since a utility function is invariant to any monotonic transformation, these subjective values and can only be estimated in terms of deviation from an arbitrary origin. When } T_0 \text{ is considered to be the base/reference/ numéraire activity,}
\]

\[
(\beta_i - \beta_0) = \gamma_0 - \gamma_i, \forall i = 1, 2, \ldots, n
\]

Special case 1: Use of \( T_0 \) with \( u_{T_0} = 0 \) as the base/reference activity implies

\[
\beta_i = \gamma_0 - \gamma_i, \forall i = 1, 2, \ldots, n
\]

Special case 2: Use of \( T_0 \) with \( \gamma_0 = 0 \) as the base/reference activity implies

\[
(\beta_i - \beta_0) = -\gamma_i,
\]

**Equalized Net (Total) Benefit of Time Use in all Activities (α_i)**

\[
\alpha_i = \beta_i + \gamma_i = \frac{U_i}{U_X} + R_i - \sum_{j=1}^{s} \frac{\partial P_j}{\partial T_i} X_j
\]

\[
= (\mu / \lambda) - R_i + \sum_{j=1}^{s} \frac{\partial P_j}{\partial T_i} X_j + R_i - \sum_{j=1}^{s} \frac{\partial P_j}{\partial T_i} X_j = (\mu / \lambda),
\]
Willingness to Pay to Reallocate Time from Activity \(i\) to \(k\) \((\rho_{i-k})\)

\[
\rho_{i-k} = \beta_i - \beta_k = (\mu/\lambda) - R_k + \sum_{j=1}^{\infty} \frac{\partial P_j}{\partial T_k} X_j - (\mu/\lambda) + R_i - \sum_{j=1}^{\infty} \frac{\partial P_j}{\partial T_i} X_j
\]

\[
\rho_{i-k} = (R_i - \sum_{j=1}^{\infty} \frac{\partial P_j}{\partial T_k} X_j) - (R_k - \sum_{j=1}^{\infty} \frac{\partial P_j}{\partial T_i} X_j) = \gamma_i - \gamma_k
\]

Factorial \(n\) possible \(\rho_{i-k}\) values can be defined for \((n+1)\) activities. However, the number can be reduced to \(n\) values, \(\rho_{i-0}\), by using \(T_0\) as the base/reference activity,

\[
\rho_{i-0} = \beta_i - \beta_0 = (R_i - R_0) + \left(\sum_{j=1}^{\infty} \frac{\partial P_j}{\partial T_0} X_j - \sum_{j=1}^{\infty} \frac{\partial P_j}{\partial T_i} X_j\right) = \gamma_i - \gamma_0
\]

Special case: When the reference activity has zero effect on availability of goods, that is \(\gamma_0 = 0\),

\[
\rho_{i-0} = \beta_i - \beta_0 = R_i - \sum_{j=1}^{\infty} \frac{\partial P_j}{\partial T_i} X_j = \gamma_i
\]

The Value of Time (VOT)

An increase in time availability can be assigned to any activity, but the net (total) benefit will be the same and equal to the VOT,

\[
VOT = \alpha_i = \frac{u_{x_i}}{u_{x_i}} + R_i - \sum_{j=1}^{\infty} \frac{\partial P_j}{\partial T_i} X_j = MRS_{i,X} + \gamma_i \quad \text{for any } i = 0, 1, \ldots, n.
\]

Since this includes a \(MRS_{i,X}\) term, which is a subjective value, the VOT cannot be estimated from observed market data. However, the assumption of marginal utility of an activity equal to zero will imply that VOT is equal to the measurable consumption effect of that activity.

4. VALUE OF TRAVEL TIME SAVING (VTT)

Commuting Time in the Simple Labour-Leisure Model

In the simplest versions of this model (Troung and Hensher, 1985; Bates, 1987; Train and McFadden, 1978), it is assumed that, while commuting requires time \((T)\) and commuting goods \((C)\), there is no direct satisfaction derived from either. In other words, neither \(T\) nor \(C\) enters the utility function but is only present in the constraint as a parameter.

1Reviews of the literature can be found in Jara-Diaz, 2000, Mackie et al., 2001, and Jiang and Marikawa, 2004.
The optimization problem can be written as

\[ \Lambda = u(X, L) + \lambda [wW + y - C - X] + \mu (M - W - L - T) \]

where \( T \) and \( C \) are parameters.

Treating \( W \) as a choice variable, the first-order conditions are

\[ \Lambda_X = u_X - \lambda = 0 \]
\[ \Lambda_L = u_L - \mu = 0 \]
\[ \Lambda_W = \lambda w - \mu = 0 \]
\[ \Lambda_\lambda = wW + y - C - X = 0 \]
\[ \Lambda_\mu = M - L - T - W = 0 \]

from which the following can be obtained

\[ \beta_L = \frac{u_L}{u_X} + \frac{\mu}{\lambda} = w, \quad \beta_W = \frac{u_W}{u_X} = 0, \quad \text{and} \quad \beta_T = \frac{u_T}{u_X} = 0 \]
\[ \alpha_L = \frac{u_L}{u_X} + \frac{\mu}{\lambda} = w, \quad \alpha_W = w, \quad \text{and} \quad \alpha_T = 0 \]
\[ \rho_{w\rightarrow L} = \frac{u_L - u_W}{\lambda} = \frac{w}{\lambda} = w, \quad \rho_{T\rightarrow L} = \frac{u_L - u_T}{\lambda} = \frac{\mu}{\lambda} = w, \quad \text{and} \quad \rho_{T\rightarrow w} = \frac{u_W - u_T}{\lambda} = 0 \]

In this formulation, commuting time and monetary cost only affect the time and budget constraints. As such, \( T = 0 \) and \( C = 0 \) would be the best choice for any individual. Given that a \( T-C \) combination must be chosen, however, the best choice would always be the least-cost one where time is valued at the wage rate, that is, minimum \((C + wT)\). To see this, after substituting for \( W \) from the time constraint into the budget constraint, one can rewrite the optimization problem as: maximize \( u = u(X, L) \) subject to \( X + wL = wM + y - (C + wT) \). The choice of \( C \) and \( T \) that minimizes the last term in the constraint would leave the maximum amount of resources for allocation between utility-generating \( X \) and \( L \).

Treating \( C \) and \( T \) as continuous variables, the best choice will correspond to a situation where the trade-off between travel time and travel cost is equal to the wage rate. Treating \( C \) and \( T \) as associated with a discrete mode choice, an individual would be better off allocating more (less) time to commuting when additional commuting time leads to a travel cost saving in excess (less) of the wage rate. The mode-choice decision is independent of preferences in this framework.

When travel time, \( T \), enters the utility function (e.g., Oort, 1969; Evans, 1972) and \( T \) is considered a choice variable, the model is equivalent to the general model presented above, where one of the activities is travel time. As we have already seen, an implication of that model is that at the optimal point there is no gain in utility from reallocation of time among non-work
activities; hence, gain from travel-time saving at the margin is zero. As Evans (1972, p. 9) noted, "... if the consumer’s allocation of time is optimal, a reduction in the time spent travelling would make him no better off (and no worse off)."

As shown in the previous section, a positive value of travel-time saving (VTTS) implies that at the optimal point the marginal utility of leisure time is greater than the marginal utility of travel time so that there is a utility gain from transferring travel time to leisure time. In the transportation economics literature, there are three different ways in which researchers derived this gap between the two marginal utilities: (1) by adding a binding minimum time constraint (DeSerpa, 1971; Evans, 1972), (2) by adding a binding minimum consumption constraint (Evans, 1972; Jara-Diaz, 2003), and (3) by introducing effective leisure time concept (Train and McFadden, 1987; Small, 1992).

**Binding Minimum Travel-Time Constraint**

DeSerpa (1971) and Evans (1972) assumed the presence of a binding minimum time requirement that prevents an individual from reaching the unconstrained optimum level of travel time. This assumption makes the marginal utility of travel time lower than the marginal utility of leisure. Reduction in travel time allows reallocation of time from the lower utility-generating activity, travel, to the higher utility-generating activity, leisure. The Lagrange multiplier corresponding to the minimum time constraint shows the difference between the marginal utility of these two time uses, which in turn equals the willingness to pay to reallocate time (VTTS). Technically, with addition of minimum travel-time constraints, given as \( T_i \geq a_i X_i \), with corresponding Lagrange multipliers, \( K_i \), DeSerpa derived the following result

\[
VTTS = \frac{u_{r_i} - u_{f_i}}{u_{x_i}} = \frac{\mu}{\lambda} \left( \frac{\mu}{\lambda} - \frac{K_i}{\lambda} \right) = \frac{K_i}{\lambda}
\]

This represents the willingness to pay to reallocate time from binding \( T_i \), travel time, to non-binding \( T_j \), leisure.

Without a minimum travel-time constraint, VTTS would be zero. In reality, it is difficult to argue that an individual is allocating the minimum required time to travel activity. In the short-run, it is always possible to reduce commuting time by adopting a faster mode, such as using a taxi instead of walking. In the long run, one can reduce travel time by changing one’s job and/or residential location. In other words, it can be argued that an individual’s choice of travel time is not consistent with a binding minimum travel-time constraint.

**Binding Minimum Consumption Constraint**

Another line of argument is that an individual is constrained to spend more than the optimal amount on travel. In addition to a minimum time constraint, Jara-Diaz (2003) introduced a binding minimum consumption constraint, given by \( C_i \geq g_i(T_i) \) where \( g_i' > 0 \). Given this constraint, Jara-Deaz derived a relationship among VTTS and the monetary value of relaxing the constraint (as well as the monetary value of other marginal utility terms).
As opposed to this view, one can view travel-time saving as allowing an individual to move closer to the utility-maximizing level of travel expenditures. As with DeSerpa’s minimum travel-time constraint, an imposed constraint is the source of positive VTTS, not the solution to an optimization problem. In most cases it is possible to reduce the monetary portion of travel costs by spending more time travelling, e.g., by walking instead of taking a taxi, although that might not be the optimal choice. The point is that how much money one spends on commuting is a choice, not constrained by some binding minimum requirement.

**Effective Leisure Time**

Most empirical studies of travel time savings are based on travel choice in a discrete-choice setting, the theoretical foundation of which was first introduced by Train and McFadden (1978). Although Train and McFadden did not emphasize VTTS, their model may be used to illustrate the approach to estimating VTTS in discrete-choice models. In discussing the value of time, Small (1992, pp. 40–43) also uses this model.

Let $u_i = u(X_i, L_i)$, where $X_i = y + wW - C_i$ and $L_i = M - W - T_i$, and where the variables are as previously defined, except that subscripted variables are defined for the $i$th mode, $i = 1, ..., n$. Maximization of $u(y + wW - C_i, M - W - T_i)$ with respect to $W$ produces a first-order condition that implies $u_{L_i} / u_{X_i} = w$. Solving the first-order condition for the utility-maximizing value of $W$ produces $W_i^* = W^*(C_i, T_i, w, y, M)$. Substituting this into the utility function gives $u^*(y + wW_i^* - C_i, M - W_i^* - T_i) = v(C_i, T_i, w, y, M)$, which is the indirect utility function that discrete-choice models estimate. Mode $i$ is chosen if and only if $v_i > v_j$ for all $j = 1, ..., n; j \neq i$. In this context, VTTS is defined as $-(\partial C / \partial T)$ for the utility-maximizing mode. Empirically, an equation such as $v_i = a_C C_i + a_T T_i + \cdots$ is estimated, so that $VTTS = -\partial C / \partial T = a_T / a_C$.

Theoretically, however, without further assumptions, $VTTS = w$ in this model, which can be seen as follows

$$VTTS = \rho_{T \rightarrow L} = \frac{u_{L_i} - u_T}{u_X} = \frac{u_{L_i}}{u_X} = w$$

or

$$\frac{\partial C}{\partial T} = \frac{u_{L_i}}{u_C} = \frac{u_{L_i}(-1)}{u_X(-1)} = \frac{u_{L_i}}{u_X} = w$$

For purposes of estimation, Train and McFadden (followed by Small) introduce the concept of effective leisure. It is the introduction of effective leisure that allows the marginal utility of leisure to be greater than the marginal utility of travel time and VTTS to be a positive fraction of the wage rate. To show this, a simplified version of effective leisure is used here.
Assuming no utility from work time and only one category of travel time, $l$, may be defined as $l = L + \theta T$, where $\theta$ is assumed to be a constant between 0 and 1.\(^2\) Then, $u = u(X, l)$, and

$$MRS_{LT} = \frac{u_L}{u_T} = \frac{u_l}{u_l \theta} = \frac{1}{\theta} > 1$$

This implies that $u_L > u_T$ and that $VTTS > 0$. Furthermore

$$VTTS = \frac{u_L - u_T}{u_X} = \frac{u_l - u_l \theta}{u_X} = (1 - \theta) \frac{u_L}{u_X} = (1 - \theta) w$$

Thus, if $\theta$ is a constant between 0 and 1, the $VTTS$ is positive and a constant fraction of the wage rate. This result is independent of the specific form of the utility function.

In conclusion, all theoretical models underlying the value of travel-time saving are based on some binding constraint on travel time and/or travel cost. None of the models addresses the question why people travel in the first place. Handy et al. (2005) and Ory et al. (2004) argued that in most cases travelling is a choice not a requirement. The next section develops and presents a model that predicts a positive $VTTS$ without imposing any minimum requirements for travel time or travel cost.

5. RESIDENTIAL LOCATION CHOICE WITH ENDOGENOUS TRAVEL TIME

This section presents a model of household choice in which residential location and travel time are simultaneously chosen without a binding minimum travel-time restriction. In the absence of such a constraint, the motivating force behind the allocation of time to travel is the reduction of housing expenditures. This is shown by incorporating a travel-distance production function into the standard residential location choice model.

The Model

The utility function. The household’s quasi-concave utility function is

$$u = u(X, H, L, W, T)$$ (4)

where $H$ is housing consumption and the other variables are as previously defined. Marginal utilities are positive for $X$, $H$, and $L$, but they may be positive or negative for $W$ and $T$.

\(^2\) In terms of Train and McFadden, let $\theta_0 = 1$ and denote $1 - \theta_j = \theta$. In terms of Small, let $\alpha_w = 0$ and denote $\alpha^k = \theta$. The assumption that $0 < \theta < 1$ ensures that travel time is not worse than work ($\theta > 0$) and not better than leisure ($\theta < 1$).
Prices. It is assumed that $P = P(K)$ and $P_K < 0$, where $K$ is distance from an urban center. This assumption does not preclude a multicentric city, only that accessibility to any center or subcenter is sufficiently desirable that housing price falls away from it. Empirical studies support this relationship (Alonso, 1964, p. 172; Muth, 1969, pp. 192, 237; Wieand, 1973; Coulson, 1991). As before, $X$ is the numeraire, so its price is unity.

The distance production function. Distance travelled depends on the amount of time spent travelling and the amount of travel goods, $X_T$, measured in terms of money

$$K = K(X_T, T)$$

where $K_i > 0$ and $K_{ii} \leq 0$. $X_T$ is assumed to have no consumption value; therefore, it is not a component of the utility function.

The budget constraint. It is assumed that an individual is free to choose to work for any number of hours at a fixed wage rate. The budget constraint states that the total expenditure on good $X$ is equal to the sum of labour and non-labour income

$$wW + y = X + P(K)H + X_T$$

The time constraint. The time constraint implies that the sum of time allocated to leisure, market work, and travel equals total available time

$$M = L + W + T$$

The optimization problem. The individual’s problem is to choose $X, H, X_T, L, T$ and $W$ to maximize (4) subject to (5), (6) and (7). After substituting (5) into (6), this problem can be set up in Lagrangean form as

$$\Lambda = u(X, H, L, W, T, \lambda) + wW + y - X - P(X_T, T)H - X_T + \mu(M - L - W - T)$$

First-order conditions for maximization are

$$\lambda_X = u_X - \lambda = 0$$
$$\lambda_H = u_H - \lambda P = 0$$
$$\lambda_L = u_L - \mu = 0$$
$$\lambda_w = u_w + \lambda w - \mu = 0$$
$$\lambda_T = u_T + \lambda P X_T - \mu = 0$$
$$\lambda_{x_T} = -\lambda(P X_T - H) + 1 = 0$$
$$\Lambda_w = wW + y - X - PK(X_T, T)H - X_T = 0$$
$$\Lambda_{\mu} = M - L - T - W = 0$$
Optimum allocation of money. Since \( \lambda > 0 \), (11) implies
\[
-P_k K_{X_T} H = 1
\]
The left-hand side of (8) shows the marginal benefit while the right hand side shows the marginal cost of a dollar spent on travel goods, \( X_T \). The marginal benefit is the reduction in housing expenditures from travelling farther and thereby obtaining a lower price of housing. The marginal cost is the additional dollar of expenditure on travel goods. This equilibrium condition therefore implies that a dollar allocated to travel goods generates a dollar’s worth of benefits in housing expenditure reduction.

From the first two FOCs we have
\[
\tilde{\lambda} = u_X = \frac{u_H}{P}
\]
Thus, the optimal allocation of income between goods \( X \) and \( H \) is such that marginal benefit from expenditure on the two goods is equalized, and that these marginal benefits are equal to the marginal utility of money, \( \lambda \).

Optimum allocation of time. From the third, fourth and fifth FOCs we have
\[
\frac{u_L}{\tilde{\lambda}} = \frac{u_W}{\tilde{\lambda}} + w = P_k K_T H = \frac{\mu}{\tilde{\lambda}}
\]
This utility maximization condition states that the marginal benefit of time spent in leisure, work, and travel is equalized and that it is equal to the value of time. In other words, at the optimal point there is no room for utility increase through reallocation of time. Note that the marginal benefit of leisure time derives solely from utility gain, \( u_L \); the marginal benefit of working time derives partly from utility gain or loss, \( u_W \), and partly from monetary gain, \( w \); and the marginal benefit of travel time derives partly from utility gain or loss, \( u_T \), and partly from monetary gain, \( P_k K_T H \), the later being the reduction in housing expenditure from travelling farther and thereby obtaining a lower housing price.

The time-allocation concepts defined above may be derived as follows
\[
\beta_L = \frac{u_L}{u_X} = \frac{\mu}{\tilde{\lambda}}, \quad \beta_W = \frac{u_W}{u_X} = \frac{\mu}{\tilde{\lambda}} - w, \quad \beta_T = \frac{u_T}{u_X} = \frac{\mu}{\tilde{\lambda}} + P_k K_T H
\]
\[
\alpha_L = \frac{u_L}{u_X} = \frac{\mu}{\tilde{\lambda}}, \quad \alpha_W = \frac{u_W}{u_X} + w = \frac{\mu}{\tilde{\lambda}}, \quad \alpha_T = \frac{u_T}{u_X} - P_k K_T H = \frac{\mu}{\tilde{\lambda}}
\]
\[
\rho_{w \rightarrow L} = \frac{u_L - u_w}{\tilde{\lambda}} = w, \quad \rho_{T \rightarrow L} = \frac{u_L - u_T}{\tilde{\lambda}} = -P_k K_T H > 0,
\]
and \( \rho_{T \rightarrow w} = \frac{u_w - u_T}{\tilde{\lambda}} = -w - P_k K_T H \)
Conclusion. Important implications of this formulation are that there is a positive VTTS from reallocation of travel time to leisure, and that VTTS is equal to an amount that can be measured from observed data without estimating an utility function.

**Housing Price Gradient and VTTS**

As noted above, the value of travel-time saving (VTTS), which is the willingness to pay to reallocate travel time to leisure, is

\[
VTTS = \rho_{T \rightarrow L} = \frac{u_L - u_T}{\lambda} = -P_K K_T H > 0
\]

As long as housing price falls with the distance between a household’s residence and its employment center, then, at the optimum, \( u_L \) must exceed \( u_T \).

The right-hand side shows the amount of money the individual can save in terms of lower housing cost by commuting one additional unit of time. At equilibrium this is the amount the individual is paying by not commuting the extra unit of time; hence, it reflects the willingness to pay to save travel time.

VTTS in this framework will, in general, vary with residential location, \( P_K \), house/lot size, \( H \), different modes due to variations in travel speed, \( K_T \), and different transportation expenditures, \( X_T \). Someone occupying a larger house, living closer to the center of employment, and/or using a faster or more expensive mode will have a higher VTTS.

An implication of this result is that the housing-price gradient in term of travel time might be a fruitful way of estimating the value of travel time saving. A hypothetical example might help explain this point. Suppose that by travelling 12 additional minutes by bus (same fixed fare) every work-day, an individual can save $20,000 on housing expenditure. Assuming a 4-percent financing cost, this saving is equivalent to a saving of $800 per year, while additional commuting time is equivalent to 50 hours per year (12 minutes times 250 working days). This gives a VTTS of $16 per hour.

Coulson (1991) has used an approach similar to this. Coulson runs a regression of housing sales price on other variables, including distance from the central business district (CBD). Using the estimated coefficient on distance of $2,633.41, the assumption of 500 one-way trips per year, and a capitalization rate of 10 percent, Coulson calculates the implied cost of transportation as \( 0.10 \times (2633.41/500) = 0.527 \) per mile. Based on data from the 1989 Statistical Abstract, he estimates the money cost of travel as $0.36 per mile. Thus, the time cost of transportation is $0.167 per mile.

**Money-Time Trade-off in Travel and VTTS**

Equation (5) implies that for any given distance, \( K \), an individual faces a trade-off between travel time, \( T \), and travel cost, \( X_T \), a mode-choice problem. The implications of this section’s model for mode choice can be examined in terms of the fifth and sixth FOCs. Dividing the second FOC by the third, using the fourth FOC, produces
\[
\frac{K_T}{K_{X_T}} = w + \frac{u_W - u_T}{\lambda}
\]

This is the standard cost-minimization condition: the negative of the slope of the “distance iso-quant” (the marginal rate of technical substitution of travel goods for travel time in distance production) is equal to the relative price of the two inputs, travel time and travel goods. The price of travel time is the wage rate plus the differential utility effect of time reallocation, while travel cost is one dollar per unit of travel goods.

The following relationship is derived from the third, fourth and fifth FOCs, and (9)

\[
\frac{K_T}{K_{X_T}} = w + \frac{u_W - u_T}{\lambda} = -P_K K_T H = \frac{u_T - u_T}{\lambda} = VTTS
\]

In addition to allowing the estimation of VTTS as \(-P_K K_T H\), this shows that one can also estimate VTTS as \(MRTS_{TX_T}\). For example, if travelling from point A to point B on a toll road entails a $4 toll but saves 15 minutes over using surface roads for the trip, then VTTS is $16 per hour. When comparing VTTS for alternative T and C combinations, it is necessary to control for origin and destination (\(P_K\)), mode (\(K_T\)), and housing characteristics (\(H\)).

6. CONCLUSIONS

The estimation of the value of travel time saving is one of the most widely studied topics in transportation economics, yet at the theoretical level it is one of the least understood. In the literature, the value-of-time concept has been used by different researchers to mean different things. Through a review of the literature on time allocation, the first part of this paper makes explicit distinctions among three concepts: the monetary value of the marginal utility of time use, the net benefits of time use, and the willingness to pay for time reallocation between two alternative activities. This paper shows that the value of time (VOT) refers to the second term, while the value of travel time saving (VTTS) refers to the last term.

The second part of the paper shows that the reason for a positive VTTS in existing models is the imposition of a minimum travel-time constraint. None of the existing models addresses the question why people travel in the first place. This paper proposes an alternative model in which there is no minimum travel-time requirement, yet there is a positive VTTS at the optimal point.

The most popular approach to estimating VTTS is based on the specification and estimation of an indirect utility function. The basis for this approach differs considerably from that underlying economic value in goods markets as understood by economists. Although people’s preferences for a particular good may differ, the market nevertheless places an economic value on the good equal to the good’s market price. Thus, it is unnecessary for the researcher to estimate a utility function to obtain the economic value of the good. The same principle can be applied to the valuation of travel-time saving. By studying market behaviour where people are observed to trade travel time for money, the researcher can estimate VTTS without estimating a util-
ity function. This paper provides two ways of estimating VTTS. One way is to estimate the variation of housing price with travel time. In this approach, VTTS will vary by mode of commuting, residential location, transport expenditures, and house size. Another way is to estimate the money-time trade-off between alternatives for travelling between two fixed points under similar travel circumstances, such as a faster, tolled expressway vs. a slower, untolled city street.
REFERENCES


